Answer on Question #62066 – Math – Differential Equations

Question

Use the power series method to obtain one solution of the following ODE:

$xy'' + y' - xy = 0,$

$x_0 = 2,\ y(x_0) = y(2) = 3,\ \ y'(x_0) = y'(2) = 0.$

Solution

$$y(x_0) = 3,\ y'(x_0) = 0,\ x_0 = 2$$

$$xy'' + y' - xy = 0$$

$$y'' = \frac{xy - y'}{x} = y - \frac{y'}{x}$$

$$y''(2) = y(2) - \frac{y'(2)}{2} = 3 - 0 = 3$$

$$y''' = \left(y - \frac{y'}{x} \right)' = y' - \left(\frac{y''}{x} - \frac{y'}{x^2}\right) = y' - \frac{y''}{x} + \frac{y'}{x^2}$$

$$y'''(2) = y'(2) - \frac{y''(2)}{2} + \frac{y'(2)}{4} = -\frac{3}{2}$$

$$y(x) = y(x_0) + \frac{y'(x_0)}{1!}(x - x_0) + \frac{y''(x_0)}{2!}(x - x_0)^2 + \frac{y'''(x_0)}{3!}(x - x_0)^3 + \ldots =$$

$$= 3 + \frac{3}{2}(x-2)^2 - \frac{3}{12}(x-2)^3 + \ldots$$

Answer: $y(x) = 3 + \frac{3}{2}(x-2)^2 - \frac{3}{12}(x-2)^3 + \cdots$