Answer on Question #62065 – Math – Differential Equations

Question

A mass of 2 kg is fixed to an end of a spring with spring constant k = 128 Nm−1 and the system is placed inside a fluid. It is set into vibration from its equilibrium position with an initial speed of 0.6 ms−1 . If the damping force due to the fluid is 40v(t) N where v is the instantaneous speed of the mass, determine the position of the mass as a function of time t.

Solution

Using Newton's second law we have:

$m\ddot{x} = -\lambda\dot{x} - kx;$

$2\ddot{x} = -40\dot{x} - 128x;$

$\ddot{x} + 20\dot{x} + 64x = 0;$ (1)

Now we need to solve characteristic equation:

$\lambda^2 + 20\lambda + 64 = 0 \Rightarrow \lambda_1 = -16, \lambda_2 = -4.$

The general solution of the differential equation (1) is

$$x(t) = C_1 e^{-16t} + C_2 e^{-4t}.$$

Now we can use the initial conditions for position and speed:

$x(t = 0) = 0 = C_1 + C_2;$

$\dot{x}(t = 0) = 0.6 = -16C_1 - 4C_2;$

$\begin{cases} C_1 + C_2 = 0 \\ -16C_1 - 4C_2 = 0.6 \end{cases} \Rightarrow \begin{cases} C_2 = -C_1 \\ -16C_1 - 4C_2 = 0.6 \end{cases} \Rightarrow \begin{cases} C_2 = -C_1 \\ 16C_2 - 4C_2 = 0.6 \end{cases} \Rightarrow \begin{cases} C_2 = -C_1 \\ 12C_2 = 0.6 \end{cases} \Rightarrow$
$\begin{cases} C_2 = -C_1 \\ C_2 = 0.6/12 \end{cases} \Rightarrow C_1 = -0.05, C_2 = 0.05;$

Position of the mass as a function of time t is

$$x(t) = -0.05e^{-16t} + 0.05e^{-4t}.$$

Answer: $x(t) = -0.05e^{-16t} + 0.05e^{-4t}.$