Answer on Question #62062 – Math – Differential Equations

Question

Solve the initial value problem

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0, y(0) = 2, y'(0) = 1 \quad (1)$$

Solution

In this question

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0 \quad (2)$$

is a second order homogeneous equation with constant coefficients.

Characteristic equation is

$$r^2 + 2r + 2 = 0$$

$$D = 2^2 - 4 \cdot 2 = -4 < 0.$$

Its roots are complex:

$$r_{1,2} = -1 \pm i.$$

The general solution of (2) is given by

$$y = C_1 e^{-x} \cos x + C_2 e^{-x} \sin x.$$

In order to find a particular solution, we use the initial conditions (1) to determine C_1 and C_2.

First, we have

$y(0) = C_1 e^0 \cos 0 + C_2 e^0 \sin 0 = C_1 = 2.$

Since

$y'(x) = -C_1 e^{-x} \cos x - C_1 e^{-x} \sin x - C_2 e^{-x} \sin x + C_2 e^{-x} \cos x,$

we get

$y'(0) = -C_1 e^0 \cos 0 - C_1 e^0 \sin 0 - C_2 e^0 \sin 0 + C_2 e^0 \cos 0 = -2 + C_2 = 1;$

hence

$C_2 = 3.$

Thus, the solution of the initial value problem (1) is

$$y = 2e^{-x} \cos x + 3e^{-x} \sin x.$$

Answer: $y = 2e^{-x} \cos x + 3e^{-x} \sin x.$