Answer on Question #62061 – Math – Differential Equations

Question

Solve the following ordinary differential equations:
i)

$$\frac{1}{x}\sin y\, dx + [(\ln x)(\cos y) + y]dy = 0$$

Solution

Let us split the left-hand side of the equation in two parts

$$\left(\frac{\sin y}{x}dx + (\ln x)(\cos y)dy\right) + ydy = 0. \quad \textbf{(1)}$$

Consider

$$\frac{\sin y}{x}dx = \sin y\, d(\ln x); \quad (\ln x)(\cos y)dy = \ln x\, d(\sin y);$$

$$ydy = d\frac{y^2}{2}.$$

Then

$$\frac{\sin y}{x}dx + (\ln x)(\cos y)dy = \sin y\, d(\ln x) + \ln x\, d(\sin y) = d(\sin y \cdot \ln x)$$

and the equation (1) is equivalent to

$$d(\sin y \cdot \ln x) + ydy = 0. \quad \textbf{(2)}$$

Integrating the formula (2) we get

$$\sin y \cdot \ln x + \frac{y^2}{2} = C,$$

where C is an arbitrary real constant.

Answer: $\sin y \cdot \ln x + \frac{y^2}{2} = C.$