

## Answer on Question #62036 – Math – Vector Calculus

### Question

For a scalar field

$$\varphi = x^n + y^n + z^n,$$

where  $n$  is a non-zero real constant,

show that

$$(\nabla\varphi, \vec{r}) = n\varphi$$

### Solution

If  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  and  $\varphi = x^n + y^n + z^n$ , then

$$\begin{aligned}\nabla\varphi &= \frac{\partial\varphi}{\partial x}\vec{i} + \frac{\partial\varphi}{\partial y}\vec{j} + \frac{\partial\varphi}{\partial z}\vec{k} = \\ &= \frac{\partial(x^n + y^n + z^n)}{\partial x}\vec{i} + \frac{\partial(x^n + y^n + z^n)}{\partial y}\vec{j} + \frac{\partial(x^n + y^n + z^n)}{\partial z}\vec{k} = \\ &= nx^{n-1}\vec{i} + ny^{n-1}\vec{j} + nz^{n-1}\vec{k};\end{aligned}$$

$$\begin{aligned}(\nabla\varphi, \vec{r}) &= (nx^{n-1}\vec{i} + ny^{n-1}\vec{j} + nz^{n-1}\vec{k}, x\vec{i} + y\vec{j} + z\vec{k}) = \\ &= nx^{n-1}x + ny^{n-1}y + nz^{n-1}z = n(x^n + y^n + z^n) = n\varphi.\end{aligned}$$