Answer on Question #61859 – Math – Vector Calculus

Question

5. Given that vectors

a→=5i−2j+3k

b→=3i+j−2k

c→=i−3j+4k,

calculate the scalar triple product

$$\vec{a} \cdot (\vec{b} \times \vec{c})$$

Solution

The scalar triple product is

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \det\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \det\begin{bmatrix} 5 & -2 & 3 \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{bmatrix}$$
$$= 5 \cdot 1 \cdot 4 + (-2) \cdot (-2) \cdot 1 + 3 \cdot 3 \cdot (-3) - 1 \cdot 1 \cdot 3 - (-3) \cdot (-2) \cdot 5 - 4 \cdot 3 \cdot (-2) =$$
$$= 20 + 4 - 27 - 3 - 30 + 24 = -12$$

Answer: -12.

Question

6. Given that vectors

a→=5i−2j+3k,

b→=3i+j−2k

and

c→=i−3j+4k,

calculate the vector triple product

$$\vec{a} \times (\vec{b} \times \vec{c})$$

Solution

$$(\vec{b} \times \vec{c}) = \det\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{bmatrix} = \hat{\imath}\begin{vmatrix} 1 & -2 \\ -3 & 4 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 3 & -2 \\ 1 & 4 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = \hat{\imath}\big(1 \cdot 4 - (-3) \cdot (-2)\big) -$$
$$j\big(3 \cdot 4 - 1 \cdot (-2)\big) + \hat{k}(3 \cdot (-3) - 1 \cdot 1) = -2\hat{\imath} - 14\hat{\jmath} - 10\hat{k}.$$

The vector triple product is

$$\vec{a} \times (\vec{b} \times \vec{c}) = \det\begin{bmatrix} i & j & k \\ 5 & -2 & 3 \\ -2 & -14 & -10 \end{bmatrix} = \hat{\imath}\begin{vmatrix} -2 & 3 \\ -14 & -10 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 5 & 3 \\ -2 & -10 \end{vmatrix} + \hat{k}\begin{vmatrix} 5 & -2 \\ -2 & -14 \end{vmatrix} =$$
$$= \hat{\imath}\big((-2)\cdot(-10) - (-14)\cdot 3\big) - j(5\cdot(-10) - (-2)\cdot 3) + \hat{k}\big(5\cdot(-14) - (-2)\cdot(-2)\big)$$
$$= 62\hat{\imath} + 44\hat{\jmath} - 74\hat{k}$$

Answer: $62\hat{\imath} + 44\hat{\jmath} - 74\hat{k}$.