

Problem #6168 What are the chances of seeing 11 or more heads in a trial of 15 unbiased coin throws? Would you reject the hypothesis that the coin is unbiased if you see 11 heads in 15 throws? Please give detailed answer.

Solution 1. The number of heads ξ in the serial of 15 throws of unbiased coin (the probability of head in each throw is 0.5) has $Bin(15, 0.5)$ distribution, that is $\mathbb{P}\{\xi = k\} = \binom{15}{k} 0.5^{15}$. And the probability in problem equals to $p = \sum_{k=11}^{15} \mathbb{P}\{\xi = k\} = 1941 \cdot 0.5^{15} \approx 0.059$.

Answer $1941 \cdot 0.5^{15} \approx 0.059$.

To verify if the coin is unbiased I recommend to use chi-squared test of goodness of fit, although the number of observation is small. The test statistics $\hat{X} = \frac{(7/2)^2}{15/2} + \frac{(7/2)^2}{15/2} = \frac{49}{15} \approx 3.26$, the critical value $x_{0.95}$ is the quantile of chi-square distribution with 1 degree of freedom (the p-value I consider is equal to 0.05) of the level 0.95 and from statistical table $x_{0.95} = 3.84$. Due to $3.26 < 3.84$ there are no reasons to reject the hypothesis that the coin is unbiased.

Answer there are no reasons to reject the hypothesis that the coin is unbiased.