Solution

Let x_n is in R and suppose that there is an M in R such that | x_n | less than or equal to M for n in N. Prove that s_n=sup{x_n, x_(n+1), ...} defines a real number for each n in N and that s_1>s_2>....

$\exists M, \forall n \in N : |x_n| \le M$ (comment $M \ge 0$) $\Rightarrow$

By definition $s = \sup\{x_n \mid n \in N\}$ follows : $-M \le \sup\{x_n\} \le M \Rightarrow s \in [-M; M] \Rightarrow s \in R$

By condition: $s_r = \sup\{x_n \mid n \in N \setminus \{1,2,3,..,r-1\}\}$

But $\forall r \in N : s_r = \sup\{x_n \mid n \in N \setminus \{1,2,3,..,r-1\}\} = s_r = \sup\{x_n \mid n \in (N \setminus \{1,2,3,..,r\}) \cup \{r\}\} =$
$= \sup\{s_{r+1}; x_r\} \ge \forall n \in N : s_{r+1} \Rightarrow s_1 \ge s_2 \ge s_3 \ge ... s_n \ge s_{n+1}$