Answer on Question #60838 – Math – Algorithms | Quantitative Methods

Question

2. a) Find by Newton's method the roots of the following equations correct to three places of decimals.
i) log .4 772393 x 10 x = near x = 6 .
ii) f (x) = x − 2sin x near x = 2 .

Solution

i) By Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Write the equation in the form $f(x) = 0$:

$$x \log_{10} x - 4.772393 = 0$$

Firstly find $f'(x)$:

$$f'(x) = \log_{10} x + \frac{1}{\ln 10}$$

Let's do the first approximation:

$$f(x_0) = 6 \cdot \log_{10} 6 - 4.772393 = -0.103485$$

$$f'(x_0) = 1.212446$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 6 - \frac{-0.103485}{1.212446} = 6.085352$$

Then do the second approximation:

$$f(x_0) = 6.085352 \cdot \log_{10} 6.085352 - 4.772393 = 0.000262$$

$$f'(x_0) = 1.21858$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 6.085352 - \frac{0.000262}{1.21858} = 6.085136$$

We see that after this approximation first three digits didn't change, so we've got it to three decimal places.

Answer: $x = 6.085$.

ii) In the second case we have

$$f(x) = x - 2\sin x$$

Similarly do all the operations like in the first one.

Find $f'(x)$:

$$f'(x) = 1 - 2\cos x$$

First approximation:

$$f(x_0) = f(2) = 2 - 2\sin 2 = 0.181405$$

$$f'(x_0) = 1 - 2\cos 2 = 1.832294$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{0.181405}{1.832294} = 1.900996$$

Second approximation:

$$f(x_1) = 1.900996 - 2 \cdot \sin 1.900996 = 0.009041$$

$$f'(x_1) = 1 - 2 \cdot \cos 1.900996 = 1.648464$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.900996 - \frac{0.009041}{1.648464} = 1.895511$$

Third approximation:

$$f(x_2) = 1.895511 - 2 \cdot \sin 1.895511 = 0.000027$$

$$f'(x_2) = 1 - 2 \cdot \cos 1.895511 = 1.638077$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.895511 - \frac{0.000027}{1.638077} = 1.895495$$

After three approximations we estimate the answer to three places of decimals.

Answer: $x = 1.895$.