Answer on Question #60736 – Math – Algebra

Question

1. Write the quadratic expression x^2 − 24x − 12 in completed square form.

Solution

It is known that

$$(a-b)^2 = a^2 - 2ab + b^2.$$

Consider

$$x^2 - 24x - 12 = x^2 - 2 \cdot 12x + 12^2 - 12^2 - 12 = (x^2 - 2 \cdot 12x + 12^2) - 12^2 - 12 =$$

$$= (x-12)^2 - 12^2 - 12 = (x-12)^2 - 156.$$

Question

2. Use the completed square form from above to solve the equation x^2 − 24x − 12 = 0, leaving your answer in exact (surd) form.

Solution

It was found in part 1 that the completed square form of the quadratic expression $x^2 - 24x - 12$ is $(x-12)^2 - 156$.

Then

$$x^2 - 24x - 12 = 0 \rightarrow (x-12)^2 - 156 = 0 \rightarrow (x-12)^2 = 156 \rightarrow x - 12 = \pm\sqrt{156} \rightarrow$$

$$\rightarrow x - 12 = \pm 2\sqrt{39} \rightarrow x = 12 \pm 2\sqrt{39}.$$

Hence solutions of the equation $x^2 - 24x - 12 = 0$ are

$$x_1 = 12 - 2\sqrt{39}$$

$$x_2 = 12 + 2\sqrt{39}$$

Question

3. Use the completed square form from part 1 to write down the vertex of the parabola

y = x^2 − 24x − 12.

Solution

The completed square form is

$$y = (x-12)^2 - 156$$

So, the vertex of the parabola is $(12, -156)$.