Answer on Question #60657 – Math – Algebra

Question

If 2-i is a solution of the equation x^2- 4x+ k= 0, what is the value of k?

Given:

$$x^2 - 4x + k = 0$$
$$x = 2 - i$$

Find:

k

Solution:

First method

$$D = 4^2 - 4k = 16 - 4k$$
$$x = \frac{4 \pm \sqrt{D}}{2} = \frac{4 \pm \sqrt{16 - 4k}}{2} = 2 \pm \sqrt{4 - k}$$

If $x = 2 - i$ is a solution of the equation $x^2 - 4x + k = 0$, then

$$2 \pm \sqrt{4 - k} = 2 - i$$
$$\Rightarrow \ \pm \sqrt{4 - k} = -i$$
$$\Rightarrow \ 4 - k = i^2$$
$$\Rightarrow \ 4 - k = -1$$
$$\Rightarrow \ k = 5$$

Second method

According to Vieta's formulas for a second-degree polynomial,

$$\begin{cases} x_1 + x_2 = 4, \\ x_1 x_2 = k, \end{cases} \quad (1)$$

where x_1, x_2 are the solutions of the equation $x^2 - 4x + k = 0$.

It is given that $x_1 = 2 - i$.

It follows from the first equation of the system (1) that

$x_2 = 4 - x_1 = 4 - (2 - i) = 2 + i$.

It follows from the second equation of the system (1) that

$k = x_1 x_2 = (2 - i)(2 + i) = 2^2 - i^2 = 4 - (-1) = 5$

Answer: $k = 5$.