Newton's method

Given a function ƒ defined over the reals x, and its derivative ƒ ', we begin with a first guess x_0 for a root of the function f. Provided the function is reasonably well-behaved a better approximation x_1 is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}\,.$$

Geometrically, $(x_1, 0)$ is the intersection with the x-axis of a line tangent to f at $(x_0, f(x_0))$.

The process is repeated as

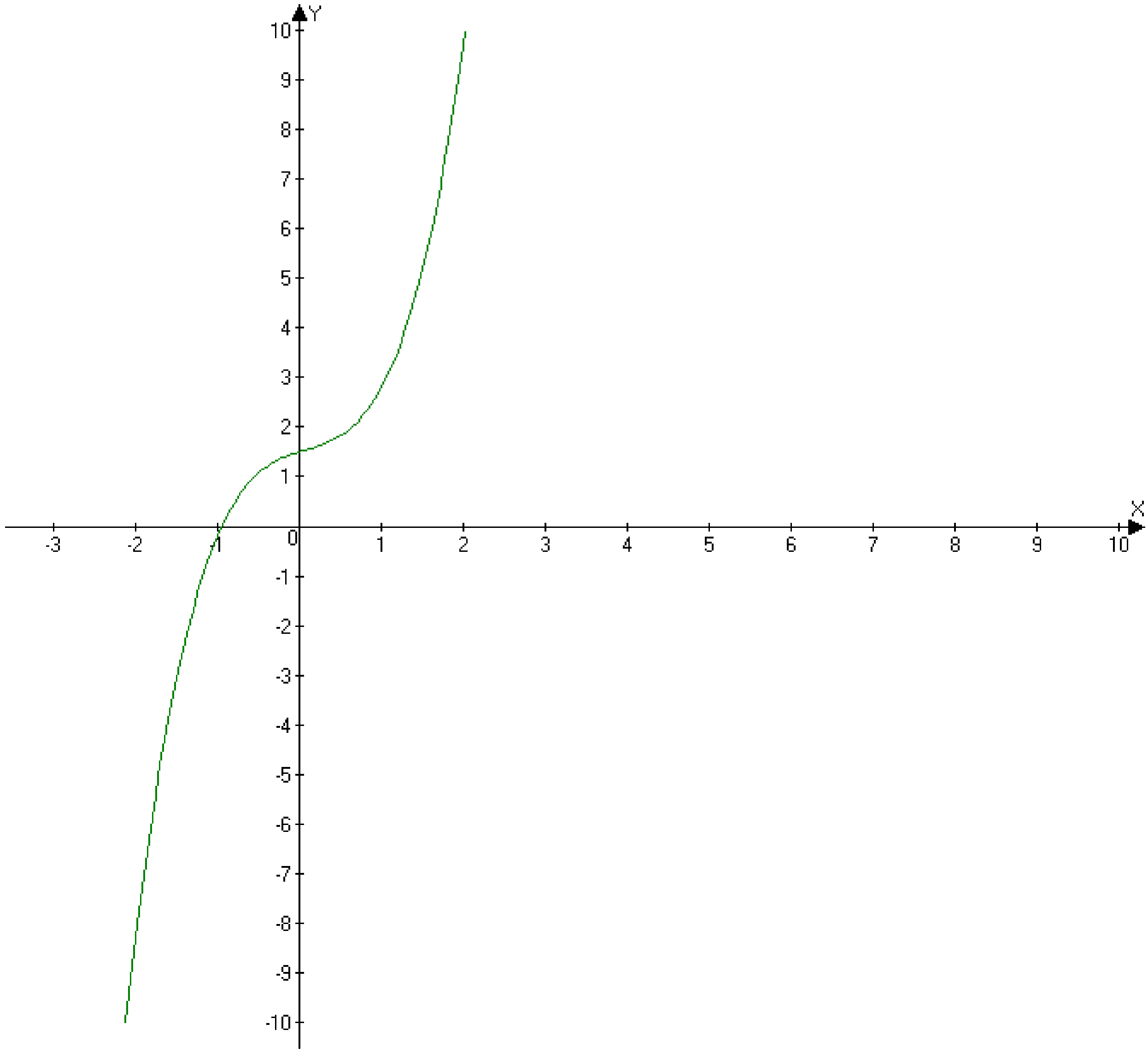
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

until a sufficiently accurate value is reached.

Example:

Given the equation: $x^3 - 0{,}2x^2 + 0{,}5x + 1{,}5 = 0$. Clarify the root with an error ε <0,001 (error, you can choose yourself if nothing is specified in the problem at this point).

$f(x) = x^3 - 0{,}2x^2 + 0{,}5x + 1{,}5$.



root is in the range [-1; 0], a = -1, b = 0.

f(-1) = -1 – 0,2 – 0,5 + 1,5 = -0,2 < 0;

f(0) = 1,5 > 0.

Find the first derivative: f'(x) = $3x^2$ *– 0,4x + 0,5.*

Find the second derivative: f''(x) = 6x – 0,4.

f''(-1) = -6 – 0,4 = -6,4 < 0;

f''(0) = – 0,4 = -0,4 < 0.

At the end of a segment [a, b] satisfies the condition f (-1) f ''(-1)> 0, so we take the initial approximation x_0 *= -1, and the calculation will be carried out by the formula*

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.$$

pre find $f'(x_0) = 3(-1)^2 - 0,4(-1) + 0,5 = 3,9.$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -1 - \frac{-0.2}{3.9} = -0.9487$$

$x_2 = -0.9466$

$x_3 = -0.9464$

If $|x_{i+1} - x_i| \le \varepsilon$*, then we have reached the desired accuracy.*

$|x_3 - x_2| \le 0.001$

$0.0002 \le 0.001$ *we have achieved the desired accuracy.*

The results of calculations by Newton's method

i	x_i	$f(x_i)$
0	-1	-0,2000
1	-0,9487	-0,0083
2	-0,9466	-0,0007
3	-0,9464	-0,0001

Answer: $x \approx -0,9464.$