Answer on Question #60030– Math – Algebra

Question

1. Sketch: $y = \frac{1}{36(x^2-6)^2(x+1)^2(x^2-3x-4)(3-2x)^3(x^2+1)(x^2-4x-1)}$

Find y-int, zeros, and multiplicities?

Solution

If

$$y(x) = \frac{1}{36(x^2-6)^2(x+1)^2(x^2-3x-4)(3-2x)^3(x^2+1)(x^2-4x-1)}$$

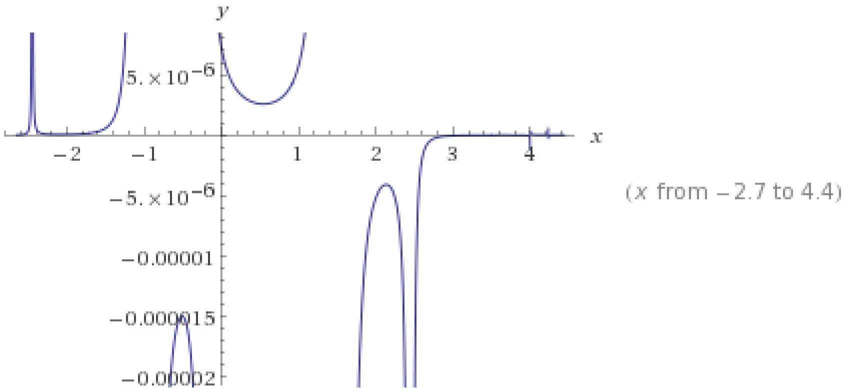
Then

the y-intercept: $\left(0, \frac{1}{36(0^2-6)^2(0+1)^2(0^2-3\cdot 0-4)(3-2\cdot 0)^3(0^2+1)(0^2-4\cdot 0-1)}\right) = (0{,}7.14 \cdot 10^{-6});$

$y \neq 0$, therefore, zeros do not exist.

Besides, $x \neq \pm\sqrt{6};\ x \neq -1; x \neq 4; x \neq 1.5; x \neq 2-\sqrt{5}; x \neq 2+\sqrt{5}.$

Sketch is given below.



If

$y(x) = \frac{1}{36}(x^2-6)^2(x+1)^2(x^2-3x-4)(3-2x)^3(x^2+1)(x^2-4x-1),$

then

$$y(x) = \frac{1}{36}\left((x-\sqrt{6})(x+\sqrt{6})\right)^2 (x+1)^2(x+1)(x-4)\cdot(-1)(2x-3)^3(x^2+1)\cdot$$

$$\cdot(x-2+\sqrt{5})(x-2-\sqrt{5}) = -\frac{8}{36}(x-\sqrt{6})^2(x+\sqrt{6})^2(x+1)^3(x-4)\left(x-\frac{3}{2}\right)^3\cdot$$

$\cdot(x^2+1)\cdot(x-2+\sqrt{5})(x-2-\sqrt{5}).$

As $x \to \infty$ the plot is similar to that of function $-\frac{8}{36}x^{15}$.

The y-intercept is

$(0, y(0)) = \left(0, -\frac{8}{36}(0-\sqrt{6})^2(0+\sqrt{6})^2(0+1)^3(0-4)\left(0-\frac{3}{2}\right)^3(0^2+1)\cdot\right.$

$\left.(0-2+\sqrt{5})(0-2-\sqrt{5})\right) = \left(0, -\frac{8}{36}\cdot 6\cdot 6\cdot 1\cdot(-4)\cdot(-1)\cdot\frac{27}{8}\cdot 1\cdot(-1)\right) = (0, 108).$

It follows from $y(x) = 0$ that $x = \sqrt{6}$ or $x = -\sqrt{6}$ or $x = -1$ or $x = 4$ or $x = \frac{3}{2}$ or $x = 2-\sqrt{5}$ or $x = 2+\sqrt{5}$.

Thus, the following values are zeros:

$x = \sqrt{6}$ (multiplicity=2),

$x = -\sqrt{6}$ (multiplicity=2),

$x = -1$ (multiplicity=3),

$x = 4$ (multiplicity=1),

$x = \frac{3}{2}$ (multiplicity=3),

$x = 2 - \sqrt{5}$ (multiplicity=1),

$x = 2 + \sqrt{5}$ (multiplicity=1).

Sketch is given below.