Answer on Question #59763 – Math – Algebra

Write the first 5 terms of the following sequences

Question

1. $a_1 = 4,$
$a_n = a_{n-1} + 4.$

Solution

Method 1

Given

$a_1 = 4.$

Putting $n = 2$ in the formula $a_n = a_{n-1} + 4$ gives

$a_2 = a_1 + 4 = 4 + 4 = 8,$

putting $n = 3$ in the formula $a_n = a_{n-1} + 4$ gives

$a_3 = a_2 + 4 = 8 + 4 = 12,$

putting $n = 4$ in the formula $a_n = a_{n-1} + 4$ gives

$a_4 = a_3 + 4 = 12 + 4 = 16,$

putting $n = 5$ in the formula $a_n = a_{n-1} + 4$ gives

$a_5 = a_4 + 4 = 16 + 4 = 20.$

Method 2

By definition, an arithmetic progression is a sequence of numbers such that the difference between the current term and the preceding term is the same for any two consecutive terms.

It follows from $a_n = a_{n-1} + 4$ that the common difference

$$d = a_n - a_{n-1} = 4$$

is the constant value of the difference between the current and the preceding consecutive terms of the arithmetic progression.

Formula for the n-th term is

$$a_n = a_1 + (n-1)d,$$

where $a_1 = 4$, $d = 4$.

Given $a_1 = 4$.

According to the formula for the n-th term,

$a_2 = a_1 + (2-1)d = a_1 + d = 4 + 4 = 8$,

$a_3 = a_1 + (3-1)d = a_1 + 2d = 4 + 2 \cdot 4 = 12$,

$a_4 = a_1 + (4-1)d = a_1 + 3d = 4 + 3 \cdot 4 = 16$,

$a_5 = a_1 + (5-1)d = a_1 + 4d = 4 + 4 \cdot 4 = 20$.

Answer:

$a_1 = 4$,

$a_2 = 8$,

$a_3 = 12$,

$a_4 = 16$,

$a_5 = 20$.

Question

2. $a_1 = -1$,
$a_n = a_{n-1} - 2$.

Solution

Method 1

Given

$a_1 = -1$.

Putting $n = 2$ in the formula $a_n = a_{n-1} - 2$ gives

$a_2 = a_1 - 2 = -1 - 2 = -3$,

putting $n = 3$ in the formula $a_n = a_{n-1} - 2$ gives

$a_3 = a_2 - 2 = -3 - 2 = -5$,

putting $n = 4$ in the formula $a_n = a_{n-1} - 2$ gives

$a_4 = a_3 - 2 = -5 - 2 = -7$,

putting $n = 5$ in the formula $a_n = a_{n-1} - 2$ gives

$a_5 = a_4 - 2 = -7 - 2 = -9$.

Method 2

By definition, an arithmetic progression is a sequence of numbers such that the difference between the current term and the preceding term is the same for any two consecutive terms.

It follows from $a_n = a_{n-1} - 2$ that the common difference

$$d = a_n - a_{n-1} = -2$$

is the constant value of the difference between the current and the preceding consecutive terms of the arithmetic progression.

Formula for the n-th term is

$$a_n = a_1 + (n-1)d,$$

where $a_1 = -1$, $d = -2$.

Given

$a_1 = -1.$

According to the formula for the n-th term,

$a_2 = a_1 + (2-1)d = a_1 + d = -1 - 2 = -3,$

$a_3 = a_1 + (3-1)d = a_1 + 2d = -1 + 2 \cdot (-2) = -5,$

$a_4 = a_1 + (4-1)d = a_1 + 3d = -1 + 3 \cdot (-2) = -7,$

$a_5 = a_1 + (5-1)d = a_1 + 4d = -1 + 4 \cdot (-2) = -9.$

Answer:

$a_1 = -1,$

$a_2 = -3,$

$a_3 = -5,$

$a_4 = -7,$

$a_5 = -9.$

Question

3. $a_1 = 0.25,$
$a_n = a_{n-1} + 0.5.$

Solution

Method 1

Given

$a_1 = 0.25.$

Putting $n = 2$ in formula $a_n = a_{n-1} + 0.5$ gives

$a_2 = a_1 + 0.5 = 0.25 + 0.5 = 0.75,$

putting $n = 3$ in formula $a_n = a_{n-1} + 0.5$ gives

$a_3 = a_2 + 0.5 = 0.75 + 0.5 = 1.25,$

putting $n = 4$ in formula $a_n = a_{n-1} + 0.5$ gives

$a_4 = a_3 + 0.5 = 1.25 + 0.5 = 1.75,$

putting $n = 5$ in formula $a_n = a_{n-1} + 0.5$ gives

$a_5 = a_4 + 0.5 = 1.75 + 0.5 = 2.25.$

Method 2

By definition, an arithmetic progression is a sequence of numbers such that the difference between the current term and the preceding term is the same for any two consecutive terms.

It follows from $a_n = a_{n-1} + 0.5$ that the common difference

$$d = a_n - a_{n-1} = 0.5$$

is the constant value of the difference between the current and the preceding consecutive terms of the arithmetic progression.

Formula for the n-th term is

$$a_n = a_1 + (n-1)d,$$

where $a_1 = 0.25$, $d = 0.5$.

Given

$a_1 = 0.25.$

According to the formula for the n-th term,

$a_2 = a_1 + (2-1)d = a_1 + d = 0.25 + 0.5 = 0.75,$

$a_3 = a_1 + (3-1)d = a_1 + 2d = 0.25 + 2 \cdot 0.5 = 1.25,$

$a_4 = a_1 + (4-1)d = a_1 + 3d = 0.25 + 3 \cdot 0.5 = 1.75,$

$a_5 = a_1 + (5-1)d = a_1 + 4d = 0.25 + 4 \cdot 0.5 = 2.25.$

Answer:

$a_1 = 0.25,$

$a_2 = 0.75,$

$a_3 = 1.25,$

$a_4 = 1.75,$

$a_5 = 2.25.$

Question

4. $a_1 = 1,$
$a_n = n \cdot a_{n-1}.$

Solution

Method 1

Given

$a_1 = 1,$

putting $n = 2$ in formula $a_n = n \cdot a_{n-1}$ gives

$a_2 = 2 \cdot a_1 = 2 \cdot 1 = 2,$

putting $n = 3$ in formula $a_n = n \cdot a_{n-1}$ gives

$a_3 = 3 \cdot a_2 = 3 \cdot 2 = 6,$

putting $n = 4$ in formula $a_n = n \cdot a_{n-1}$ gives

$a_4 = 4 \cdot a_3 = 4 \cdot 6 = 24,$

putting $n = 5$ in formula $a_n = n \cdot a_{n-1}$ gives

$a_5 = 5 \cdot a_4 = 5 \cdot 24 = 120.$

Method 2

Given

$a_1 = 1.$

Consider

$$a_n = n \cdot a_{n-1} = n \cdot \big((n-1) \cdot a_{n-2}\big) = n(n-1)a_{n-2} = n(n-1) \cdot \big((n-2) \cdot a_{n-3}\big) =$$

$$= n(n-1)(n-2)a_{n-3} = \cdots = n(n-1)(n-2) \dots \big(n-(n-2)\big)a_{n-(n-1)} =$$
$$= n(n-1)(n-2) \dots 2a_1 = n(n-1)(n-2) \dots 2 \cdot 1 = n!$$

according to the definition of factorial.

Then using the formula $a_n = n!$ for the n-th term compute

$a_2 = 2! = 1 \cdot 2 = 2,$

$a_3 = 3! = 1 \cdot 2 \cdot 3 = 6,$

$a_4 = 4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24,$

$a_5 = 5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120.$

Answer:

$a_1 = 1,$

$a_2 = 2,$

$a_3 = 6,$

$a_4 = 24,$

$a_5 = 120.$