Answer on Question #59139 – Math – Algebra

Question

Solve the simultaneous equation

$$\begin{cases}\log_2 x - \log_2 y = 2, \\ \log_2(x - 2y) = 3.\end{cases}$$

Solution

$$\begin{cases}\log_2 x - \log_2 y = 2 \\ \log_2(x - 2y) = 3\end{cases} \Leftrightarrow \begin{cases}\log_2 \frac{x}{y} = 2 = \log_2 4 \\ \log_2(x - 2y) = 3 = \log_2 8\end{cases} \Leftrightarrow \begin{cases}\frac{x}{y} = 4 \\ x - 2y = 8\end{cases} \Leftrightarrow \begin{cases}x = 4y \\ x - 2y = 8\end{cases}$$

$$\Leftrightarrow \begin{cases}x = 4y \\ 4y - 2y = 8\end{cases} \Leftrightarrow \begin{cases}x = 4y \\ 2y = 8\end{cases} \Leftrightarrow \begin{cases}x = 4y \\ y = 4\end{cases} \Leftrightarrow \begin{cases}x = 16 \\ y = 4\end{cases}$$

Answer: $x = 16; y = 4.$

Question

A projectile is launched with a velocity of $30ms^{-1}$. If it just clear a barrier 40 m high at a distance 30 m away from the point of projection, calculate, correct to one decimal place:

- Angle of projection
- Horizontal range
- Maximum height attained (Take $g = 10ms^{-2}$)

Solution

Equation of projectile flight:

$$y = -\frac{q}{2v_0^2 \cos^2 \alpha} x^2 + x \operatorname{tg} \alpha$$

Because $\cos^2 \alpha = \frac{1}{\operatorname{tg}^2 \alpha + 1}$, then $y = -\frac{q(\operatorname{tg}^2 \alpha + 1)}{2v_0^2} x^2 + x \operatorname{tg} \alpha$.

According to the condition, point $(x, y) = (30; 40)$, $v_0 = 30$, $q = 10$. Then:

$$40 = -\frac{10(\operatorname{tg}^2 \alpha + 1)}{2 \cdot 30^2} \cdot 30^2 + 30 \operatorname{tg} \alpha$$

$$40 = -5(\operatorname{tg}^2 \alpha + 1) + 30 \operatorname{tg} \alpha$$

$$5(\operatorname{tg}^2 \alpha - 6 \operatorname{tg} \alpha + 9) = 0$$

$$5(\operatorname{tg}\alpha - 3)^2 = 0$$

$$\operatorname{tg}\alpha = 3$$

Angle of projection $\alpha = \arctan 3 = \tan^{-1}(3) = 1.25rad = 71°34'$

So, equation of path is

$$40 = -\frac{10(3^2+1)}{2 \cdot 30^2} \cdot x^2 + 3x$$

$$y = -\frac{1}{18}x^2 + 3x$$

Horizontal range ($y = 0$) is defined by the following equation:

$$-\frac{1}{18}x^2 + 3x = 0$$

$$x\left(-\frac{1}{18}x + 3\right) = 0$$

$$x_1 = 0 \qquad\qquad -\frac{1}{18}x + 3 = 0$$

$$x = 3 \div \frac{1}{18}$$

$$x_2 = 54$$

Thus, horizontal range is $s = 54m$.

The function $y = -\frac{1}{18}x^2 + 3x$ attains its maximum at the point, where

$$x_h = \frac{-3}{2 \cdot \left(-\frac{1}{18}\right)} = \frac{54}{2} = 27;$$

$$y(x_h) = y(27) = -\frac{1}{18} \cdot 27^2 + 3 \cdot 27 = \frac{81}{2} = 40.5.$$

Maximum height attained is $h_{max} = 40.5m$.

Answer:

angle of projection is $\alpha = \arctan 3 \approx 1.2rad \approx 71°34'$;

horizontal range is 54 m;

maximum height attained is $h = 40.5$ m.

Question

Express $\left(5-2\sqrt{10}\right)/\left(3\sqrt{5}+\sqrt{2}\right)$ in the form $m\sqrt{2}+n\sqrt{5}$.

Solution

$$\frac{5-2\sqrt{10}}{3\sqrt{5}+\sqrt{2}}=m\sqrt{2}+n\sqrt{5}$$

$$\left(m\sqrt{2}+n\sqrt{5}\right)\cdot\left(3\sqrt{5}+\sqrt{2}\right)=5-2\sqrt{10}$$

$$3m\sqrt{10}+2m+15n+n\sqrt{10}=5-2\sqrt{10}$$

$$(3m+n)\sqrt{10}+(2m+15n)=5-2\sqrt{10}$$

$$\begin{cases}2m+15n=5\\3m+n=-2\end{cases}\Leftrightarrow\begin{cases}2m+15(-3m-2)=5\\n=-3m-2\end{cases}\Leftrightarrow\begin{cases}-43m=35\\n=-3m-2\end{cases}\Leftrightarrow\begin{cases}m=-\dfrac{35}{43}\\n=-3\cdot\left(-\dfrac{35}{43}\right)-2\end{cases}$$

$$\Leftrightarrow\begin{cases}m=-\dfrac{35}{43}\\n=\dfrac{19}{43}\end{cases}$$

Answer: $-\frac{35}{43}\sqrt{2}+\frac{19}{43}\sqrt{5}$, so $m=-\frac{35}{43}, n=\frac{19}{43}$.

Question

$3x^2+2y^2+xy+x-7=0$. Find dy/dx at the point $(-2,1)$

Solution

$$\frac{dy}{dx}=-\frac{F_x'}{F_y'}$$

$$F_x'=6x+y+1,\quad F_y'=4y+x$$

$$\frac{dy}{dx}=-\frac{6x+y+1}{4y+x}$$

$$\left.\frac{dy}{dx}\right|_{x=-2,y=1}=-\frac{6\cdot(-2)+1+1}{4\cdot1+(-2)}=-\frac{-10}{2}=5$$

Answer: 5.