Answer on Question #58423 – Math – Vector Calculus

Question

1. Find the angle between
A=2x+2j−k
and
B=6i−3j+2k

600
450
690
790

Solution

If *A*=2x+2j−k and *B*=6i−3j+2k, then
$|A| = \sqrt{2^2 + 2^2 + (-1)^2} = 3, |B| = \sqrt{6^2 + (-3)^2 + 2^2} = 7.$
$(A, B) = 2 \cdot 6 + 2 \cdot (-3) + (-1) \cdot 2 = 4.$
$\cos\phi = \frac{(A,B)}{|A| \cdot |B|} = \frac{4}{21}.$
$\phi = arccos\frac{4}{21} \approx 79^{\circ}.$
Answer: $arccos\frac{4}{21} \approx 79^{\circ}.$

Question

2. Determine the value of a so that
A=2i+aj+k
and
B=4i−2j−2k
are perpendicular

a=5
a=3
a=1
a=7

Solution

If *A*=2i+aj+k and *B*=4i−2j−2k are perpendicular, then
$0 = (A, B) = 2 \cdot 4 + a \cdot (-2) + 1 \cdot (-2) = 8 - 2a - 2 = 6 - 2a \Rightarrow\ 6 - 2a = 0 \Rightarrow$
$\Rightarrow 3 - a = 0 \Rightarrow a = 3.$
Answer: $a = 3.$

Question

3. Determine a unit vector perpendicular to the plane of
A=2i−6j−3k

and
B=4i+3j−k

±(37i−27j+67k)
±(35i+25j−65k)
±(14i−34j−12k)
±(−23i−13j+34k)

Solution

The equation of the plane is
$\begin{vmatrix} x & 2 & 4 \\ y & -6 & 3 \\ z & -3 & -1 \end{vmatrix} + D = 6x + 6z - 12y + 24z + 9x + 2y + D = 15x - 10y + 30z + D.$
$n = (15, -10,30)$ is a vector perpendicular to the plane,
$|n| = \sqrt{15^2 + (-10)^2 + 30^2} = \sqrt{225 + 100 + 900} = \sqrt{1225} = 35.$
A unit vector perpendicular to the plane is
$n_1 = \frac{n}{|n|} = \left(\frac{15}{35}, -\frac{10}{35}, \frac{30}{35}\right) = (\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}),$
hence $n_1 = \frac{3}{7}i - \frac{2}{7}j + \frac{6}{7}k.$
Answer: $\frac{3}{7}i - \frac{2}{7}j + \frac{6}{7}k.$

Question

4. Find the work done in moving an object along a vector
r=3i+2j−5k

3
5
7
9

Solution

The vector $\bar{F} = F_1 i + F_2 j + F_3 k$ is needed.
Then work done in moving an object along a vector r=3i+2j−5k is
$A = (\bar{F}, \bar{r}) = 3F_1 + 2F_2 - 5F_3.$

Question

5. Given that
A=2i−j+3k
and
B=3i+2j−k,
find
A·B

3
6
1
9

Solution

Given that *A*=2i−j+3k and *B*=3i+2j−k,
$A \cdot B = 2 \cdot 3 + (-1) \cdot 2 + 3 \cdot (-1) = 1.$
Answer: $A \cdot B$ =1.

Question

6. If
A=2i−3j−k
and
B=i+4j−2k,
find
(A+B)×(A−B)

3i+4j+25k
2i+6j+2k
−20i−6j−22k
−3i−5j−25k

Solution

If *A*=2i−3j−k and *B*=i+4j−2k,
$A + B = (3,1,-3)$, $A - B = (1,-7,1).$
$(A+B) \times (A-B) = \begin{vmatrix} i & j & k \\ 3 & 1 & -3 \\ 1 & -7 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -3 \\ -7 & 1 \end{vmatrix} i - \begin{vmatrix} 3 & -3 \\ 1 & 1 \end{vmatrix} j + \begin{vmatrix} 3 & 1 \\ 1 & -7 \end{vmatrix} k =$
$\big(1 \cdot 1 - (-7) \cdot (-3)\big) - \big(3 \cdot 1 - 1 \cdot (-3)\big)j + (3 \cdot (-7) - 1 \cdot 1)k = -20i - 6j - 22k.$
Answer: $(A+B) \times (A-B) = -20i - 6j - 22k.$

Question

7. If
A=3i−j+2k,
B=2i+j−k,
and
C=i−2j+2k,
find
(A×B)×C

15i+15j−5k

5i+5j−5k

−10i+10j−5k

15i+10j−5k

Solution

If *A*=3i−j+2k, *B*=2i+j−k, and *C*=i−2j+2k,

$A \times B = \begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix} = i \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} - j \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} + k \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} = \big((-1)\cdot(-1) - 1\cdot 2\big)i -$

$-j(3\cdot(-1) - 2\cdot 2) + k\big(3\cdot 1 - 2\cdot(-1)\big) = -i + 7j + 5k.$

$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -1 & 7 & 5 \\ 1 & -2 & 2 \end{vmatrix} = i \begin{vmatrix} 7 & 5 \\ -2 & 2 \end{vmatrix} - j \begin{vmatrix} -1 & 5 \\ 1 & 2 \end{vmatrix} + k \begin{vmatrix} -1 & 7 \\ 1 & -2 \end{vmatrix} = (7\cdot 2 - (-2)\cdot 5)i -$

$-\big((-1)\cdot 2 - 1\cdot 5\big)j + \big((-1)\cdot(-2) - 1\cdot 7\big)k = 24i + 7j - 5k.$

Answer: $(A \times B) \times C = 24i + 7j - 5k.$

Question

8. Determine a unit vector perpendicular to the plane of

A=2i−6j−3k

and

B=4i+3j−k

35i−25j+65

17i−37j+47

37i−27j+67

27i−47j+57

Solution

The equation of the plane is

$\begin{vmatrix} x & 2 & 4 \\ y & -6 & 3 \\ z & -3 & -1 \end{vmatrix} + D = 6x + 6z - 12y + 24z + 9x + 2y + D = 15x - 10y + 30z + D.$

A vector perpendicular to the plane is $n = (15, -10, 30)$,

$|n| = \sqrt{225 + 100 + 900} = \sqrt{1225} = 35.$

A unit vector perpendicular to the plane is

$n_1 = \frac{n}{|n|} = \left(\frac{15}{35}, -\frac{-10}{35}, \frac{30}{35}\right) = (\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}),$

hence

$n_1 = \frac{3}{7}i - \frac{2}{7}j + \frac{6}{7}k.$

Answer: $n_1 = \frac{3}{7}i - \frac{2}{7}j + \frac{6}{7}k.$

Question

9. Evaluate

(2i−3j)·[(i+j−k)×(3i−k)]

4

5
6
8

Solution

$(i+j-k)\times(3i-k)=\begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}=-i-2j-3k.$

$(2i-3j)\cdot[(i+j-k)\times(3i-k)]=(2i-3j)\cdot(-i-2j-3k)=2\cdot(-1)+(-3)\cdot(-2)+$
$+0\cdot(-3)=-2+6+0=4.$

Answer: 4 **.**

Question

10. If
A=i−2j−3k,
B=2i+j−k
and
C=i+3j−2k,
evaluate
(A×B)·C

-25
11
15
-20

Solution

$A\times B=\begin{vmatrix} i & j & k \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix}=\begin{vmatrix} -2 & -3 \\ 1 & -1 \end{vmatrix}i-\begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix}j+\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}k=\big((-2)\cdot(-1)-1\cdot(-3)\big)i-$
$\big(1\cdot(-1)-2\cdot(-3)\big)j+\big(1\cdot1-2\cdot(-2)\big)k=5i-5j+5k.$

$(A\times B)\cdot C=(5i-5j+5k)\cdot(i+3j-2k)=5\cdot1+(-5)\cdot3+5\cdot(-2)=5-15-10=$
$=-20.$

Answer: $(A\times B)\cdot C=-20.$