Answer on Question #58320 – Math – Complex Analysis

Question

The polar form of the complex number

$$z = x + iy$$

is given by

Solution

The algebraic form of the complex number z is $z = x + iy$.
The polar form of the complex number z is given by

$$z = A(cos\varphi + isin\varphi).$$

Relationship between the polar and algebraic forms of a complex number:

$$A = \sqrt{x^2 + y^2},$$

$$\varphi = \begin{cases} arccos\dfrac{x}{\sqrt{x^2+y^2}}, & y \geq 0, \sqrt{x^2+y^2} > 0, \\ -arccos\dfrac{x}{\sqrt{x^2+y^2}}, & y < 0, \sqrt{x^2+y^2} > 0, \\ undefined, & \sqrt{x^2+y^2} = 0; \end{cases}$$

or

$$\varphi = \begin{cases} arctan\dfrac{y}{x}, & x > 0, \\ +\dfrac{\pi}{2}, & x = 0, y > 0, \\ -\dfrac{\pi}{2}, & x = 0, \quad y < 0, \\ arctan\dfrac{y}{x} + \pi, & x < 0, y \geq 0, \\ arctan\dfrac{y}{x} - \pi, & x < 0, y < 0. \end{cases}$$

Relationship between the algebraic and polar forms of a complex number:

$$x = Acos\varphi, y = Asin\varphi.$$