Find the derivative of cot square root x using first principle. Please show your work

Solution

$$\frac{d\left(\cot\sqrt{x}\right)}{dx} = \lim_{h\to 0}\frac{\cot\sqrt{x+h}-\cot\sqrt{x}}{h} = \lim_{h\to 0}\frac{\left(\frac{1}{tan\left(\sqrt{h+x}\right)}-\frac{1}{tan\left(\sqrt{x}\right)}\right)}{h}$$

Indeterminate form of type 0/0. Applying L'Hospital's rule we have,

$$\lim_{h\to 0}\frac{\left(\frac{1}{tan\left(\sqrt{h+x}\right)}-\frac{1}{tan\left(\sqrt{x}\right)}\right)}{h} = \lim_{h\to 0}\frac{\frac{d}{dh}\left(\frac{1}{tan\left(\sqrt{d+x}\right)}-\frac{1}{tan\left(\sqrt{x}\right)}\right)}{\frac{dh}{dh}}$$

$$\frac{d}{dh}\left(\frac{1}{tan\left(\sqrt{d+x}\right)}-\frac{1}{tan\left(\sqrt{x}\right)}\right) = -\frac{1}{2\sqrt{d+x}\sin(\sqrt{d+x})^2}$$

Thus:

$$\lim_{h\to 0}\frac{-\frac{1}{\tan\left(\sqrt{x}\right)}+\frac{1}{\tan\left(\sqrt{h+x}\right)}}{h} = \lim_{h\to 0} -\frac{1}{2\sqrt{h+x}\sin\left(\sqrt{d+x}\right)^2} = -\frac{1}{2}\left(\lim_{h\to 0}\frac{1}{\sqrt{h+x}\sin\left(\sqrt{h+x}\right)^2}\right)$$

$$= -\frac{1}{2\sqrt{\lim_{h\to 0}(h+x)}\left(\lim_{h\to 0}\sin\left(\sqrt{h+x}\right)^2\right)} = -\frac{1}{2\sqrt{x}\sin\left(\sqrt{\lim_{h\to 0}(h+x)}\right)^2} = -\frac{\csc^2(\sqrt{x})}{2\sqrt{x}}$$

Answer:

$$\frac{d\left(\cot\sqrt{x}\right)}{dx} = \lim_{h\to 0}\frac{\cot\sqrt{x+h}-\cot\sqrt{x}}{h} = -\frac{\mathbf{csc^2}(\sqrt{\boldsymbol{x}})}{\mathbf{2}\sqrt{\boldsymbol{x}}}$$