Answer on Question #58301 – Math – Algorithms | Quantitative Methods

Question

Use newton's method to approximate SQRT of 11 to 5 decimal places.

Solution

Finding *SQRT(S)* is the same as solving the equation

$$x^2 - S = 0 \qquad (1)$$

Therefore, any general numerical root-finding algorithm can be used for solwing (1). Let $f(x) = x^2 - S$, $f'(x) = 2x$. In Newton's method we use the following recurrent equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^2 - S}{2x_n}$$

$$x_{n+1} = \frac{x_n + \frac{S}{x_n}}{2} \qquad (2)$$

Using condition, here $S = 11$, $n = 0,1,2,3,\ldots$ is a number of iteration step, and x_n is the approximation for square root on the n-th step of the iterative process.
Substitute for $S = 11$ in formula (2):

$$x_{n+1} = \frac{x_n + \frac{11}{x_n}}{2} \qquad (3)$$

Choose the zero-approximation, and let it be $x_0 = 3.2 < \sqrt{11}$ for instance.

Therefore, the calculations for next approximations will be following:

1. $x_1 = \dfrac{3.2 + \frac{11}{3.2}}{2} \approx 3.31875$
2. $x_2 = \dfrac{3.31875 + \frac{11}{3.31875}}{2} \approx 3.31663$
3. $x_3 = \dfrac{3.31663 + \frac{11}{3.31663}}{2} \approx 3.31662$

The difference between x_3 and x_2 is less or equal to 0.00001. Thus, the required accuracy is attained and x_3 is the appropriate approximation

$$SQRT(11) = \sqrt{11} \approx 3.31662$$

Answer: $\sqrt{11} \approx 3.31662$