Answer on Question #57668 - Math - Analytic Geometry

Question

1. Find the general equation of the line parallel to 4x - 3y = 15 and passing:
   a. at a distance 2 from the origin
   b. twice as far from the origin
   c. 2 units far from the origin
   d. at a distance 5 from the given line.

Solution:

Given the line: $4x - 3y = 15$

It is equal to

$4x - 3y - 15 = 0$ or $y = \frac{4}{3}x - 5$

Lines which are parallel to given line have the same angular coefficient $k = \frac{4}{3}$.

And their equation is $y = \frac{4}{3}x + b$, where b is the intercept or $\frac{4}{3}x - y + b = 0$.

In the case of a line in the plane given by the equation $ax + by + c = 0$, where a, b and c are real constants with a and b not both zero, the distance from the line to a point (x_0, y_0) is

$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2+b^2}}$. Let use this formula for the next calculations.

a) At a distance 2 from the origin.

$d = 2$ from (0;0)

Substitute the relevant numbers into the formula for the distance:

$$2 = \frac{\left|\frac{4}{3} * 0 - 1 * 0 + b\right|}{\sqrt{\frac{16}{9} + 1}}$$

$$|b| = \frac{10}{3}$$

$b = \frac{10}{3}$ or $b = -\frac{10}{3}$

Substitute b into $y = \frac{4}{3}x + b$.

Answer: $y = \frac{4}{3}x \pm \frac{10}{3}$.

b) Twice as far from the origin.

Given line: $4x - 3y - 15 = 0$

Point: (0;0)

Find the distance from a given line to the origin with our formula:

$$d = \frac{|4 * 0 - 3 * 0 - 15|}{\sqrt{16 + 9}} = 3.$$

Twice as far from the origin: $d_1 = 2 * d = 2 * 3 = 6$.

$\frac{4}{3}x - y + b = 0$.

$d_1 = 6$.

Point: (0;0).

Substitute the relevant numbers into the formula for the distance:

$6 = \frac{\left|\frac{4}{3}*0-1*0+b\right|}{\sqrt{\frac{16}{9}+1}}$

$|b| = 10$

$b = 10$ or $b = -10$.

Substitute b into $y = \frac{4}{3}x + b$.

Answer: $y = \frac{4}{3}x \pm 10$.

c) 2 units far from the origin.

$d_1 = d + 2 = 3 + 2 = 5$.

Point (0;0).

$\frac{4}{3}x - y + b = 0$.

Substitute the relevant numbers into the formula for the distance:

$$5 = \frac{\left|\frac{4}{3} * 0 - 1 * 0 + b\right|}{\sqrt{\frac{16}{9} + 1}}$$

$$|b| = \frac{25}{3}$$

$b = \frac{25}{3}$ or $b = -\frac{25}{3}$

Substitute b into $y = \frac{4}{3}x + b$.

Answer: $y = \frac{4}{3}x \pm \frac{25}{3}$.

d) At a distance 5 from the given line.

At a distance $d_1 = d + 5 = 3 + 5 = 8$ from the origin.

Point: (0;0).

$\frac{4}{3}x - y + b = 0$.

Substitute the relevant numbers into the formula for the distance:

$$8 = \frac{\left|\frac{4}{3} * 0 - 1 * 0 + b\right|}{\sqrt{\frac{16}{9} + 1}}$$

$|b| = \frac{40}{3}$.

$b = \frac{40}{3}$ or $b = -\frac{40}{3}$.

Substitute b into $y = \frac{4}{3}x + b$.

Answer: $y = \frac{4}{3}x \pm \frac{40}{3}$.

Question

2.

Find the general equation of the line parallel to x + y = 3 and passing:

a. at a distance 2 from the origin

b. 2 square root of 2 units farther from the origin

c. 2/3 as far from the origin

d. at a distance 3 square root of 2 from the given line

Solution

Given the line: $x + y = 3$.

It is equal to

$x + y - 3 = 0$ or $y = -x + 3$

Lines which are parallel to the given line have the same slope $k = -1$.

Their equation is $y = -x - b$, where *b* is the intercept, hence obtain $x + y + b = 0$.

In the case of a line in the plane given by the equation *ax* + *by* + *c* =0 , where *a*, *b* and *c* are real constants with *a* and *b* not simultaneously zero, the distance from the line to a point (x_0,y_0) is

$d = \frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$. Let use this formula for the next calculations.

a) At a distance 2 from the origin.

$d = 2$ from (0;0).

$x + y + b = 0$.

Substitute the relevant numbers into the formula for the distance:

$$2 = \frac{|1 * 0 + 1 * 0 + b|}{\sqrt{1 + 1}}$$

$$|b| = 2\sqrt{2}$$

$$b = \pm 2\sqrt{2}$$

Substitute b into $y = -x - b$.

Answer: $y = -x \pm 2\sqrt{2}$.

b) 2 square root of 2 units farther from the origin.

Given line: $x + y - 3 = 0$

Point: (0;0)

Find the distance from the given line to the origin with the formula:

$$d = \frac{|1*0+1*0-3|}{\sqrt{1+1}} = \frac{3}{\sqrt{2}}.$$

$d_1 = d + 2\sqrt{2} = \frac{3}{\sqrt{2}} + 2\sqrt{2} = \frac{7}{\sqrt{2}}.$

$x + y + b = 0$.

$d_1 = \frac{7}{\sqrt{2}}.$

Point: (0;0).

Substitute the relevant numbers into the formula for the distance:

$$\frac{7}{\sqrt{2}} = \frac{|1*0+1*0+b|}{\sqrt{1+1}}$$

$$|b| = 7$$

$$b = \pm 7$$

Substitute b into $y = -x - b$.

Answer: $y = -x \pm 7$.

c) 2/3 as far from the origin.

$$d_1 = \frac{2}{3}d = \frac{2}{3} * \frac{3}{\sqrt{2}} = \sqrt{2}$$

$x + y + b = 0$.

$d_1 = \sqrt{2}$.

Point: (0;0).

Substitute the relevant numbers into the formula for the distance:

$$\sqrt{2} = \frac{|1*0+1*0+b|}{\sqrt{1+1}}$$

$$|b| = 2$$

$$b = \pm 2$$

Substitute b into $y = -x - b$.

Answer: $y = -x \pm 2$.

d) At a distance 3 square root of 2 from the given line.

$$d_1 = 3\sqrt{2} + d = 3\sqrt{2} + \frac{3}{\sqrt{2}} = \frac{9}{\sqrt{2}}$$

$x + y + b = 0$.

$d_1 = \frac{9}{\sqrt{2}}.$

Point: (0;0).

Substitute the relevant numbers into the formula for the distance:

$$\frac{9}{\sqrt{2}}. = \frac{|1*0+1*0+b|}{\sqrt{1+1}}$$

$$|b| = 9$$

$$b = \pm 9$$

Substitute b into $y = -x - b$.

Answer: $y = -x \pm 9$.

Question

3.

Find the general equation of the line:

a. parallel to the line 2x + 3y = 6 and passing at a distance 5 square root of 13 over 13 from the point (-1, 1)

b. parallel to the line x - y + 9 = 0 and passing at a distance 5 square root of 2 from the point (1, 4)

c. perpendicular to the line 3x + 4y = 7 and passing at a distance 4 from the point (1, -2)

d. perpendicular to the line 3x - 4y = 20 and passing at a distance 2 from the point (-1, 1)

Solution:

a) $2x + 3y = 6$.

$$y = \frac{6 - 2x}{3}$$
$$y = -\frac{2}{3}x + 2$$
$$k = -\frac{2}{3}$$

$y = -\frac{2}{3}x + b$ or $-\frac{2}{3}x - y + b = 0$.

Point: (-1,1).

$d = 5\sqrt{13}$.

$-\frac{2}{3}x - y + b = 0$.

Substitute the relevant numbers into the formula for the distance:

$$5\sqrt{13} = \frac{\left|-1 * \left(-\frac{2}{3}\right) - 1 + b\right|}{\sqrt{\frac{4}{9} + 1}}$$
$$b = 27$$

Substitute b into $y = -\frac{2}{3}x + b$

Answer: $y = -\frac{2}{3}x + 27$.

b) Parallel to the line x - y + 9 = 0 and passing at a distance 5 square root of 2 from the point (1, 4).

$$y = x + 9$$
$$k = 1$$

$y = x + b$ or $x - y + b = 0$.

Point: (1,4).

$d = 5\sqrt{2}$.

$x - y + b = 0.$

Substitute the relevant numbers into the formula for the distance:

$$5\sqrt{2} = \frac{|1 * 1 - 1 * 4 + \mathrm{b}|}{\sqrt{1+1}}$$

$$|-3 + b| = 10$$

$b = 13$ or $b = -7$

Substitute b into $y = x + b$.

Answer: $y = x + 13$ or $y = x - 7$.

c) Perpendicular to the line 3x + 4y = 7 and passing at a distance 4 from the point (1, -2).

If lines are perpendicular then product of their slopes is equal to -1.

In the given line:

$$y = -\frac{3}{4}x + \frac{7}{4}$$

$$k = -\frac{3}{4}$$

So, the perpendicular line has a slope of $k_1 = -\frac{1}{k} = -\frac{1}{-\frac{3}{4}} = \frac{4}{3}$.

Equation of a perpendicular line is

$$y = \frac{4}{3}x + b$$

or

$\frac{4}{3}x - y + b = 0.$

Point: (1,-2).

$d = 4.$

Substitute the relevant numbers into the formula for the distance:

$$4 = \frac{\left|1 * \frac{4}{3} + 2 * 1 + \mathrm{b}\right|}{\sqrt{\frac{16}{9} + 1}}$$

$b = \frac{10}{3}$ or $b = -10.$

Substitute b into $y = \frac{4}{3}x + b$.

Answer: $y = \frac{4}{3}x + \frac{10}{3}$ or $y = \frac{4}{3}x - 10.$

d) Perpendicular to the line 3x - 4y = 20 and passing at a distance 2 from the point (-1, 1).

If lines are perpendicular then product of their slopes is equal to -1.

In the given line:

$$y = \frac{3}{4}x - 5$$

$$k = \frac{3}{4}$$

So, $k_1 = -\frac{1}{k} = -\frac{1}{\frac{3}{4}} = -\frac{4}{3}$.

$y = -\frac{4}{3}x + b.$

$-\frac{4}{3}x - y + b = 0.$

Point: (-1,1).

$d = 2.$

Substitute the relevant numbers into the formula for the distance:

$$2 = \frac{\left|-\frac{4}{3} * (-1) - 1 + \mathrm{b}\right|}{\sqrt{\frac{16}{9} + 1}}$$

$b = -\frac{11}{3}$ or $b = 3$.

Substitute b into $y = -\frac{4}{3}x + b$.

Answer: $y = -\frac{4}{3}x - \frac{11}{3}$ or $y = -\frac{4}{3}x + 3$.