

Answer on Question #57582 – Math – Analytic Geometry

Question

CD is trisected at points P and Q. Find the position vectors of points of trisection, if the position vectors of C and D are \vec{c} and \vec{d} respectively.

Let O be the origin.

Given:

$$\begin{aligned}\overline{OC} &= \vec{c} & CP &= PQ = QD \\ \overline{OD} &= \vec{d}\end{aligned}$$

Find:

$$\begin{aligned}\overline{OP} &= \vec{p} \\ \overline{OQ} &= \vec{q}\end{aligned}$$

Solution

$$\overline{CD} = \vec{d} - \vec{c}$$

On the one hand,

$$\begin{aligned}\overline{CP} &= \vec{p} - \vec{c} \\ \overline{CQ} &= \vec{q} - \vec{c}\end{aligned}$$

On the other hand,

$$\begin{aligned}\overline{CP} &= \frac{1}{3}(\vec{d} - \vec{c}) \\ \overline{CQ} &= \frac{2}{3}(\vec{d} - \vec{c})\end{aligned}$$

$$\vec{p} = \vec{c} + \overline{CP} = \vec{c} + \frac{1}{3}(\vec{d} - \vec{c}) = \frac{2}{3}\vec{c} + \frac{1}{3}\vec{d}$$

$$\vec{q} = \vec{c} + \overline{CQ} = \vec{c} + \frac{2}{3}(\vec{d} - \vec{c}) = \frac{1}{3}\vec{c} + \frac{2}{3}\vec{d}$$

$$\text{Answer: } \vec{p} = \frac{2}{3}\vec{c} + \frac{1}{3}\vec{d}, \quad \vec{q} = \frac{1}{3}\vec{c} + \frac{2}{3}\vec{d}.$$