#5746 Build the linear operator on $\mathbb{R}^4$ that does not possess eigenvectors
Solution It is obvious that in order a linear operator on $\mathbb{R}^4$ not to possess eigenvectors it is neccessary and sufficient for operator only to have complex eigenvalues with non-zero imaginary part.

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

A posses two eigenvalues of multiplicity 2, they are $\lambda_{1,2} = 1 \pm i$(it is a consequence of that A has block form), thus the equality $A\mathbf{v} = \lambda_{1,2}\mathbf{v}$, $\mathbf{v} \neq 0$ is impossible.