Answer on Question #57357 – Math – Analytic Geometry

Question

Write the equation of the directrix of the conic section shown below. Write your answer without using spaces.

$$y^2 + 16y + 4x + 4 = 0. \quad (*)$$

Solution

By definition the canonical equation of a parabola with horizontal axis of symmetry has the form

$$y^2 = 2px, \quad (1)$$

where *p* is the distance from the vertex to the focus and the vertex to the directrix. The equation of the directrix for parabola (1) has the form

$$x = -\frac{p}{2}. \quad (2)$$

The vertex of parabola (1) is at the point (0, 0).

The conic section (*) is a parabola. Let`s show it. Rewriting (1) in more convenient form we obtain

$$y^2 + 16y + 64 - 64 + 4x + 4 = 0$$

$$(y + 8)^2 + 4x - 60 = 0$$

$$(y + 8)^2 = -4 \cdot (x - 15). \quad (3)$$

As we see, the equation (3) describes the parabola with the vertex at the point (15,-8).

Let`s introduce the new coordinates

$$y' = y + 8, \;\; x' = x - 15. \quad (4)$$

Then the equation (3) takes the form

$$(y')^2 = -4x'. \quad (5)$$

Comparing (2) and (5) we can write

$$2p = -4 \Longrightarrow p = -2.$$

Thus, the equation of the directrix in the new system of coordinates (4) is

$$x' = -\frac{-2}{2} = 1$$

$$x' = 1. \quad (6)$$

Therefore, taking into account (4) and (6), we can write the directrix equation in the basic system of coordinates:

$$1 = x - 15$$

$$x = 16. \qquad (7)$$

Answer: $x = 16$.