Answer on Question #57356 - Math – Analytic Geometry

Question 1. What are the coordinates of the vertices of the conic section shown below?

$$\frac{(y+2)^2}{16} - \frac{(x-3)^2}{9} = 1$$

A: $(0;\ -2)$ and $(6;\ -2)$

B: $(-2;\ -1)$ and $(-2;\ 7)$

C: $(-1;\ -2)$ and $(7;\ -2)$

D: $(3;\ -6)$ and $(3;\ 2)$

Solution

This conic section is a hyperbola.

The equation of a hyperbola is:

$$\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = \pm 1$$

So, we have "North–South opening hyperbola".

The coordinates of the center are $(3;\ -2)$. Then, the coordinates of the vertices of the hyperbola are $(3;\ -2 \pm b) \Leftrightarrow (3;\ -2 \pm 4) \Leftrightarrow (3;\ -6)$ and $(3;\ 2)$.

Answer: D. $(3;\ -6)$ and $(3;\ 2)$.

Question 2. What are the coordinates of the foci of the conic section shown below?

$$\frac{(y+2)^2}{25} - \frac{(x-3)^2}{4} = 1$$

A: $\left(3 \pm \sqrt{21};\ -2\right)$

B: $\left(3 \pm \sqrt{29};\ -2\right)$

C: $\left(3;\ -2 \pm \sqrt{29}\right)$

D: $\left(3;\ -2 \pm \sqrt{21}\right)$

Solution

This conic section is a hyperbola.

The equation of a hyperbola is

$$\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = \pm 1$$

So, we have "North–South opening hyperbola".

We have: $a = 2$ and $b = 5$. The coordinates of the center are $(3;\ -2)$. Therefore

$c = \sqrt{a^2 + b^2} = \sqrt{29}$. And the coordinates of the foci of the hyperbola are $(3;\ -2 \pm c) \Leftrightarrow (3;\ -2 \pm \sqrt{29})$.

Answer: C $\left(3;\ -2 \pm \sqrt{29}\right)$.

Question 3. Which direction does the graph of the equation shown below open?

$$x^2 + 6x - 4y + 5 = 0$$

A: up

B: down

C: right

D: left

Solution

Transform the expression:

$$x^2 + 6x - 4y + 5 = x^2 + 6x + 9 - 9 - 4y + 5 = (x + 3)^2 - 4y + 14 = 0$$

So, we have:

$$(x + 3)^2 = 4y - 14$$

The graph of the equation opens up.

Answer: A up.