Question

1)   Which of the following correctly identifies the vertices that lie on the major axis of the conic section shown below?

$$\frac{(x+2)^2}{64} + \frac{(y-1)^2}{81} = 1$$

A: $(-2, -7)$ and $(-2, 9)$

B: $(-10, 1)$ and $(6, 1)$

C: $(-11, 1)$ and $(7, 1)$

D: $(-2, -8)$ and $(-2, 10)$

Solution

We know that the standard equation of the ellipse is as follows:

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \text{ where } 0 < b < a \text{ (vertical major axis)}$$

In this case $h = -2, \ k = 1, \ b = 8, \ and \ a = 9$.

Now, we can find the requested information as follows:

Coordinates of the vertices: $(h, k + a)$ and $(h, k - a)$, that is,

$$(-2, 10) \text{ and } (-2, -8)$$

Answer: D: $(-2, -8)$ and $(-2, 10)$.

Question

2)   Which of the following expresses the coordinates of the foci of the conic section shown below?

$$\frac{(x-2)^2}{4} + \frac{(y+5)^2}{9} = 1$$

A: $(2 \pm \sqrt{13}, -5)$

B: $(2, -5 \pm \sqrt{5})$

C: $(2 \pm \sqrt{5}, -5)$

D: $(2, -5 \pm \sqrt{13})$

Solution

We know that the standard equation of the ellipse is as follows:

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \ 0 < b < a \text{ (vertical major axis)}$$

In this case $h = 2, \ k = -5, \ b = 2, \ and \ a = 3$. Then

$$c = \sqrt{a^2 - b^2},$$

$$c = \sqrt{9 - 4} = \sqrt{5}$$

Coordinates of the Foci: $(h, k + c)$ and $(h, k - c)$, that is, $(2, -5 + \sqrt{5})$ and $(2, -5 - \sqrt{5})$.

Answer: B: $(2, -5 \pm \sqrt{5})$.

Question

3) What is the length of the transverse axis of the conic section shown below?

$$\frac{(y + 2)^2}{25} - \frac{(x - 3)^2}{4} = 1$$

Solution

The line going from one vertex, through the center, and ending at the other vertex is called the "transverse" axis. In this case the transverse axis is vertical (in other words, the center, foci, and vertices line up above and below each other, parallel to the y-axis).

Equation is in the standard form of a hyperbola

$$\frac{(y - h)^2}{a^2} - \frac{(x - k)^2}{b^2} = 1$$

whose transverse axis is $y - axis$, and its length equals $2a$.

In this task it is $2 \cdot 5 = 10$.

Answer: 10.