Answer on Question #57064 – Math – Combinatorics | Number Theory

Question

What is the result of 0 power of 0 ?

Solution

In fact, 0^0 is not defined.

Nevertheless, there is an agreement in mathematics that $0^0 = 1$ like $0! = 1$. It is also accustomed in many different calculators and software applications.

These arguments arise in algebra.

On the one hand, it is true that $0^0 = 1$, because by the definition of power:

$$a^n = 1 \cdot \underbrace{a \cdot \ldots \cdot a}_{n\ times}.$$

When a=0 and n=0 we have:

$$0^0 = 1 \cdot \underbrace{0 \cdot \ldots \cdot 0}_{0\ times} = 1.$$

On the other hand, it is true that $0^0 = 0$:

$$0^x = 0^{1+x-1} = 0^1 \cdot 0^{x-1} = 0 \cdot 0^{x-1} = 0,$$

which is true since anything times 0 is 0. That means that

$$0^0 = 0$$

The next arguments arise in calculus.

On the one hand, the limit of x^x as x tends to zero from the right is 1. In other words, if we want the x^x function to be right continuous at 0, we should define it to be 1.

On the other hand, the function $f(x, y) = y^x$ has a discontinuity at the point $(x, y) = (0,0)$.

In particular, when we approach (0,0) along the line with $x = 0$ we get

$$\lim_{y \to 0} f(0, y) = 1$$

But when we approach $(0,0)$ along the line segment with $y = 0$ and $x > 0$ we get

$$\lim_{x \to 0^+} f(x, 0) = 0.$$

Therefore, the value of $\lim_{(x,y) \to (0,0)} f(x, y)$ depends on the direction that we take the limit. This means that there is no way to define 0^0 that will make the function y^x continuous at the point $(x, y) = (0,0)$.