Answer on Question #56855 – Math – Algorithms | Quantitative Methods

For the given functions f (x), let x0 = 0, x1 = 0.6, and x2 = 0.9. Construct interpolation polynomials of degree at most one and at most two to approximate f (0.45), and find the absolute error.

a. f (x) = cos x

Solution

$$y_0 = f(x_0) = \cos 0 = 1;$$

$$y_1 = f(x_1) = \cos 0.6 = 0.8253;$$

$$y_2 = f(x_2) = \cos 0.9 = 0.6216;$$

a) The Lagrange interpolation polynomial of degree at most 1 is constructed as follows:

$$L_1(x) = \begin{cases} \frac{x_1 - x}{x_1 - x_0} y_0 + \frac{x - x_0}{x_1 - x_0} y_1, for\ x \in [x0, x1], \\ \frac{x_2 - x}{x_2 - x_1} y_1 + \frac{x - x_1}{x_2 - x_1} y_2, for\ \mathrm{x} \in [\mathrm{x1}, \mathrm{x2}], \end{cases}$$

$$L_1(x) = \begin{cases} 1 - 0.2912x, for\ x \in [0, 0.6], \\ 1.2327 - 0.679x, for\ x \in [0.6, 0.9], \end{cases}$$

$f(0.45) \approx 0.8690$ is approximated through the linear Lagrange polynomial: $f(0.45) = \cos 0.45 = 0.9004$.

The absolute error is

$$|0.9004 - 0.8690| = 0.0314.$$

b) The Lagrange interpolation polynomial of degree at most 2 is constructed as follows:

$$L_2(x) = l_0(x)y_0 + l_1(x)y_1 + l_2(x)y_2,$$

where

$$l_0(x) = \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right)$$

$$l_1(x) = \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right)$$

$$l_2(x) = \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right)$$

$f(0.45) \approx 0.8980$ is approximated through the quadratic Lagrange polynomial.

The absolute error is

$$|0.9004 - 0.8980\ | = 0.0024.$$