

Answer on Question #56771 – Math – Combinatorics | Number Theory

Find the number of three-digit numbers from 100 to 999 inclusive which have any one digit that is the average of the other two.

Solution

The 4 sets ($\{102\}$, $\{204\}$, $\{306\}$, $\{408\}$) of numbers include a zero and they have only 4 possible combinations because if the number begins with 0 it will be less than 100:

102, 120, 201, 204, 210, 240, 306, 360, 402, 408, 420, 480, 603, 630, 804 and 840.

The 9 sets ($\{111\}$, $\{222\}$, ..., $\{999\}$) of numbers consists of the identical elements and they admit only 1 combination because it will be the same number no matter which way it goes.

Next, there are 16 sets of three different numbers that don't include a zero left. All sets will admit 6 combinations.

16 sets:

$\{123, 135, 147, 159, 234, 246, 258, 345, 357, 369, 456, 468, 567, 579, 678, 789\}$

For example, the set $\{123\}$ admits 6 possibilities: 123, 132, 213, 231, 312, 321.

Overall, we have

$$4 \times 4 = 16$$

and

$$9 \times 1 = 9$$

and

$$16 \times 6 = 96$$

Therefore, the total number of three-digit numbers from 100 to 999 inclusive which have any one digit that is the average of the other two will be

$$16 + 9 + 96 = 121.$$

Answer: 121.