Answer on Question #56761 – Math – Combinatorics | Number Theory

How many 15-letter arrangements of 5 *A*'s, 5 *B*'s and 5 *C*'s have no *A*'s in the first 5 letters, no *B*'s in the next 5 letters, and no *C*'s in the last 5 letters?

Solution

We have the following different cases, in each of them we use the formula $\overline{P_{n=k_1+k_2}}(k_1, k_2) = \frac{n!}{k_1!\cdot k_2!}$:

1) The first 5 letters are 5 *C*'s. Then the next 5 letters may be only all *A*'s, and the last 5 letters may be only all *B*'s. We have $(\overline{P_5}(5))^3 = 1$ arrangement.

2) In the first 5 letters there is only 1 letter *B* (other 4 letters may be only *C*'s). Then in the second 5 letters there are 1 *C* and 4 *A*'s, and in the last 5 letters there are 1 *A* and 4 *B*'s. Total number of the different arrangements for this cases is equal to $(\overline{P_5}(4,1))^3 = 5^3 = 125.$

3) In the first 5 letters there is only 2 letters *B* (other 3 letters may be only *C*'s). Then in the second 5 letters there are 2 *C*'s and 3 *A*'s, and in the last 5 letters there are 2 *A*'s and 3 *B*'s. Total number of the different arrangements for this case is equal to $(\overline{P_5}(3,2))^3 = 10^3 = 1000$.

Other cases are the following:

4) In the first 5 letters there is only 3 letters *B* (other 2 letters may be only *C*'s).

5) In the first 5 letters there is only 4 letters *B* (another letter may be only *C*).

6) In the first 5 letters there are 5 *B*'s.

Cases 4)-6) will be symmetric to cases 3)-1).

Therefore, we have $(1 + 125 + 1000) \cdot 2 = 1126 \cdot 2 = 2252$ arrangements.

Answer: 2252.