Answer on Question #56759 – Math – Combinatorics | Number Theory

For each positive integer *k*, let Sk denote the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is *k*. For example, S3 is the sequence 1, 4, 7, 10,…
Find the number of values of *k* for which Sk contains the term 361.

Solution

Since S_k is increasing arithmetic sequence of integers whose first term is 1, then a_n which belongs to S_k can be represented as $a_n = 1 + kn$ (in this case $1 = a_0$), where n is integer. Thus, there exists an integer p such that $a_p = 1 + kp = 361$, hence $kp = 361 - 1 = 360$.

Let's find possible values of k. Since k and p positive integers (it is obvious that $p \neq 0$) and $360 = kp$, then k and p are divisors of 360, and $360 = 1 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$.

Thus, $k = 2^i \cdot 3^j \cdot 5^t$, where $0 \leq i \leq 3,$, $0 \leq j \leq 2,$ $,0 \leq t \leq 1,$ i, j, t are integer. Therefore, we obtain that there are $4 \cdot 3 \cdot 2 = 24$ different combinations of integers i, j, t. Thus, the number of values of *k*, for which S_k contains the term 361, is 24.

Answer: 24.