

Answer on Question #56707 – Math – Combinatorics | Number Theory

Calculate the total number of words, taking all letters at a time, such that at least one repeating letter is at odd position, each word contains letters from the word MATHEMATICS.

Solution

The repeating letters in the word MATHEMATICS are A (2 times), T (2 times) and M (2 times), letters H, E, I, C and S appear in our word only once. In total there are 11 letters, hence 11 positions at which we are to arrange our letters to obtain new words. So there are 6 odd positions – these are positions with numbers 1, 3, 5, 7, 9 and 11. Since there are only 5 letters which appear only once, no matter which arrangement of the given letters from the word MATHEMATICS we choose, one of the repeating letters will be on the odd position, because one of the odd positions will never be filled by the letter which doesn't repeat (there are 6 odd positions to fill and only 5 letters which do not repeat). So the number of words satisfying the given conditions is in fact equal to the total number of words we can form out of the given letters taking all letters at a time. Recall that if we have n objects, and the i -th object must appear in the sequence m_i times (so that there are $m_1 + \dots + m_n$ objects in the sequence in total), there are $\frac{(m_1 + \dots + m_n)!}{m_1! \dots m_n!}$ possible sequences.

So in our case $n = 8$, $m_1 = m_2 = m_3 = m_4 = m_5 = 1$ (these correspond to letters H, E, I, C, S),
 $m_6 = m_7 = m_8 = 2$ (these correspond to letters A, M, T). So the total number of possible words is

$\frac{11!}{2!2!2!} = \frac{11!}{6}$, which is the desired answer according to our previous arguments .

Answer: $\frac{11!}{6}$.