7. What is the maximum number of relative extrema contained in the graph of this function?

f(x) = 3x^5 − x^3 + 4x − 2

Solution

1. $f(x) = 3x^5 - x^3 + 4x - 2$
2. $f'(x) = 15x^4 - 3x^2 + 4$

   Let x^2 = t >0

   $15t^2 - 3t + 4 = 0$

   D =9 -4*15*4<0, coefficient $15 > 0$.
3. Thus, $f'(x) = 15x^4 - 3x^2 + 4 > 0$ for all real x.

   Besides, the first derivative of the function exists in the set of real numbers and does not vanish, the function has no relative extreme points.

Answer: 0.

8. In which direction does the left side of the graph of this function point?

f(x) = 3x^3 − x^2 + 4x − 2

Solution

1. $f(x) = 3x^3 - x^2 + 4x - 2$
2. $f'(x) = 9x^2 - 2x + 4$

   D=4- 4*9*4<0, coefficient 9>0.
3. Thus, $f'(x) = 9x^2 - 2x + 4 > 0$ for all real x.

   Besides, the first derivative of the function exists in the entire set of real numbers and does not vanish, the function has no relative extreme points. As $f'(x) > 0$ for all real x, the function is monotonically increasing from left to right.

Answer: Up.

9. In which direction does the right side of this graph point?

f(x) = 3x^3 − x^2 + 4x − 2

Solution

1. $f(x) = 3x^3 - x^2 + 4x - 2$
2. $f'(x) = 9x^2 - 2x + 4$

   D=4-4*9*4<0
3. Thus, $f'(x) = 9x^2 - 2x + 4 > 0$ for all real x.

   Besides, the first derivative of the function exists in the entire set of real numbers and does not vanish, the function has no extreme points.

   As $f'(x) > 0$ for all real x, the function is monotonically increasing from left to right.

Answer: Up.