Answer on Question #56246 – Math – Calculus

A man is walking from his front door to his car. The length of garden from his door to the road is 4 m. The car is parked 10 m up the road. Because of snow, he can only walk at 0.4 m/s through the garden, and 0.5 m/s on the road. What route should he take to minimize the time taken to get to his car?

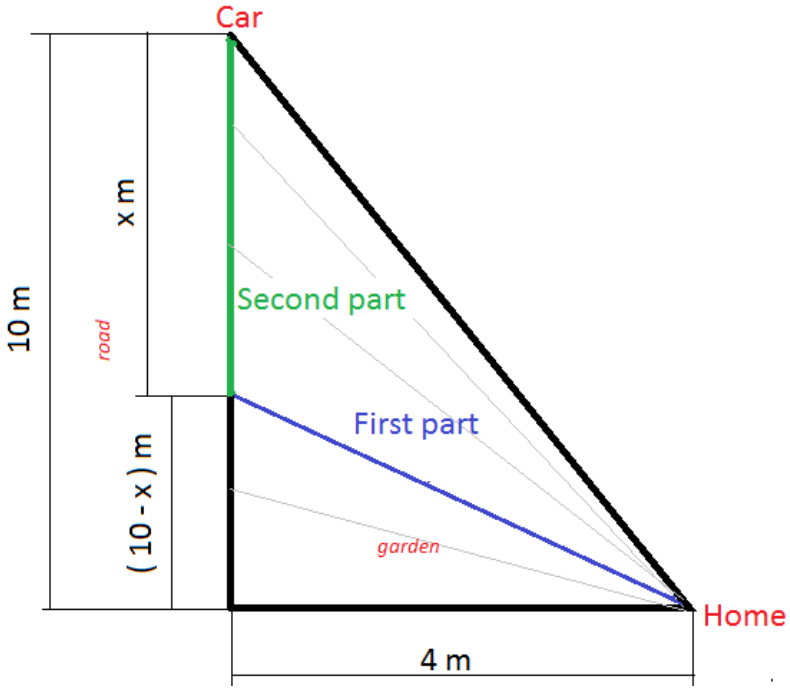
Set it up like this: Let x be the from the end of the path to the point on the road that he aims for.

Then find expressions for each of the two parts of his journey, and use the corresponding speeds to determine the time taken for each part.

Find an expression for the total time and minimize it.

Solution

$$0 \le x \le 10$$



Time taken for the first part:

$t1 = S1/v1,$

$S1 = \sqrt{4^2 + (10-x)^2} \quad m,$

$v1 = 0.4m/s,$

$$t1 = \frac{\sqrt{4^2 + (10-x)^2}}{0.4} \quad s$$

Time taken for the second part:

$t2 = S2/v2,$

$S2 = x \quad m,$

$v2 = 0.5 \quad m/s,$

$$t2 = \frac{x}{0.5} = 2x \quad s$$

Total time:

$$T = t1 + t2 \to min$$

$$T = \frac{\sqrt{4^2 + (10-x)^2}}{0.4} + 2x$$

$$T' = \frac{-20 + 2x}{2 \cdot 0.4\sqrt{116 - 20x + x^2}} + 2 = \frac{2.5x - 25}{\sqrt{x^2 - 20x + 116}} + 2$$

$\frac{2.5x-25}{\sqrt{x^2-20x+116}} + 2 = 0,$

$\frac{2.5x-25+2\sqrt{x^2-20x+116}}{\sqrt{x^2-20x+116}} = 0,$

$2.5x - 25 + 2\sqrt{x^2 - 20x + 116} = 0,$

$(25 - 2.5x)^2 = 4(x^2 - 20x + 116),$

$2.25x^2 - 45x + 161 = 0,$

$$\begin{bmatrix} x_1 = \dfrac{45 + \sqrt{2025 - 1449}}{4.5} \\ x_2 = \dfrac{45 - \sqrt{2025 - 1449}}{4.5} \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 = \dfrac{69}{4.5} \\ x_2 = \dfrac{21}{4.5} \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 = \dfrac{46}{3} > 10 \\ x_2 = \dfrac{14}{3} \end{bmatrix}$$

Take the value of $x = \frac{14}{3}$, which is less than 10.

Minimized total time:

$$T = \frac{\sqrt{4^2 + (10 - \frac{14}{3})^2}}{0.4} + 2\frac{14}{3} = 26\ s.$$

Answer: 26 seconds.