Answer on Question #56240 – Math – Calculus

Question

1. Suppose a patient is given a continuous intravenous infusion of glucose at a constant rate of mg/min. Then, the rate at which the amount of glucose in the bloodstream is changing at time (in minutes) because of this infusion is given by A'(t)=re^-at mg/min, where is a positive constant associated with the rate at which excess glucose is eliminated from the bloodstream and is dependent on the patient's metabolism rate.
a) Derive an expression for the amount of glucose in the bloodstream at time t if A(0)=0
b) If and mg/min, find the amount of glucose in the bloodstream 5 minutes after treatment began.

Solution

If we have the formula of the rate of glucose in the bloodstream at time t

$$A'(t) = re^{-at},$$

we can find the mass of of glucose in the bloodstream at time t, considering that $A'(t) = \frac{dA}{dt}$,

$$A(t) = \int A'(t)dt = \int re^{-at}dt = -\frac{r}{a}e^{-at} + C$$

a) Let's us consider condition $A(0) = 0$:

$$A(0) = -\frac{r}{a}e^0 + C = C - \frac{r}{a} = 0,$$

hence

$$C = \frac{r}{a}.$$

So the final answer is

$$A(t) = \frac{r}{a}(1 - e^{-at})$$

b) Using formula $A(t) = \frac{r}{a}(1 - e^{-at})$, we substitute $t = 5\ min$ and obtain

$$A(5) = \frac{r}{a}(1 - e^{-5a})$$

Answer:

a) $A(t) = \frac{r}{a}(1 - e^{-at})$,
b) $A(5) = \frac{r}{a}(1 - e^{-5a})$.