Answer on Question #56190 – Math – Calculus

Find the area of the largest rectangle with one corner at the origin, the opposite corner in the first quadrant on the graph of the parabola f(x)=48−4x^2, and sides parallel to the axes.

Solution

Two sides of the rectangle are equal to *x* and *y = f(x)* = 48 – 4x^2. Hence its area is equal to *S=48x-4*x^3. It has the maximum value for *x* satisfying the equation

$$\frac{dS}{dx} = 48 - 12x^2 = 0.$$

Consequently, *x*=2 since the rectangle is located in the first quadrant.

$\frac{d^2S}{dx^2} = -24x,\ \left.\frac{d^2S}{dx^2}\right|_{x=2} = -24 \cdot 2 = -48 < 0$, therefore *x*=2 is indeed a local maximum.
Thus, the largest area is equal to $S = 48 \cdot 2 - 4 \cdot 2^3 = 64$.

Answer: 64.