Answer on Question #56189 – Math – Calculus

The manager of a large apartment complex knows from experience that 120 units will be occupied if the rent is 396 dollars per month. A market survey suggests that, on the average, one additional unit will remain vacant for each 9 dollar increase in rent. Similarly, one additional unit will be occupied for each 9 dollar decrease in rent. What rent should the manager charge to maximize revenue?

Solution

Revenue is found by multiplying number of units N by the rent of each unit r.

$$R = Nr$$

If there are n increases of 9 dollars each in the rent, then

$$r = 396 + 9n$$

and

$$N = 120 - n$$

And so,

$$R = (120 - n)(396 + 9n) = 47520 + 684n - 9n^2$$

with respect to n, the maximum revenue occurs when $\frac{dR}{dn} = 0$, because $\frac{d^2R}{dn^2} = -18 < 0$.

But

$$\frac{dR}{dn} = 684 - 18n = 0 \rightarrow n = 38$$

This means that for 38 increases each of 9 dollars in the rent, revenue is maximum.

The rent is then

$$r = 396 + 9(38) = \$738.$$

If there are n decreases of 9 dollars each in the rent, then

$$r = 396 - 9n$$

and

$$N = 120 + n$$

And so,

$$R = (120 + n)(396 - 9n) = 47520 - 684n - 9n^2$$

With respect to n, the maximum revenue occurs when $\frac{dR}{dn} = 0$

But,

$$\frac{dR}{dn} = -684 - 18n = 0 \rightarrow n = -38$$

The rent is then

$$r = 396 - 9(-38) = \$738.$$

Thus, the manager should charge $\$738$ of the rent to maximize revenue.