Answer on Question #56129 – Math – Vector Calculus

If $\boldsymbol{A} = x^{z^3}\boldsymbol{i} - 2^{x^2}yz\boldsymbol{j} + 2y^{z^4}\boldsymbol{k}$, find $\nabla \times \boldsymbol{A}$ at point $(1, -1, 1)$.

Solution

Vector (cross) product $\nabla \times \boldsymbol{A}$ can be rewritten in matrix form and computed:

$$\nabla \times \boldsymbol{A} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^{z^3} & -2^{x^2}yz & 2y^{z^4} \end{vmatrix} = \left(\frac{\partial}{\partial y}2y^{z^4} + \frac{\partial}{\partial z}2^{x^2}yz\right)\boldsymbol{i} - \left(\frac{\partial}{\partial x}2y^{z^4} - \frac{\partial}{\partial z}x^{z^3}\right)\boldsymbol{j} +$$

$$+\left(-\frac{\partial}{\partial x}2^{x^2}yz - \frac{\partial}{\partial y}x^{z^3}\right)\boldsymbol{k} = \left(2z^4y^{z^4-1} + 2^{x^2}y\right)\boldsymbol{i} + \left(3z^2x^{z^3}\ln x\right)\boldsymbol{j} - \left(xyz2^{x^2+1}\ln 2\right)\boldsymbol{k}.$$

Now substituting point $(1, -1, 1)$ into the expression for $\nabla \times \boldsymbol{A}$:

$$\nabla \times \boldsymbol{A}_{\,(1,-1,1)} = (2-2)\boldsymbol{i} + (3\ln 1)\boldsymbol{j} - (-4\ln 2)\boldsymbol{k} = 4\ln 2\,\boldsymbol{k}.$$

Answer:

$$\nabla \times \boldsymbol{A}_{\,(1,-1,1)} = 4\ln 2\,\boldsymbol{k}.$$