

Answer on Question #56126 – Math – Vector Calculus

Let

$$A = x^2y\mathbf{i} - 2xz\mathbf{j} + 2yz\mathbf{k}$$

, find $\text{Curl curl } A$.

$$3\mathbf{j} + 4\mathbf{k}$$

$$2x + 2\mathbf{k}$$

$$(2x + 2)\mathbf{j}$$

$$3\mathbf{j} - 4\mathbf{k}$$

Solution

$$\begin{aligned} \text{curl } A &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & -2xz & 2yz \end{vmatrix} = \mathbf{i} \left(\frac{\partial}{\partial y} (2yz) - \frac{\partial}{\partial z} (-2xz) \right) - \mathbf{j} \left(\frac{\partial}{\partial x} (2yz) - \frac{\partial}{\partial z} (x^2y) \right) + \mathbf{k} \left(\frac{\partial}{\partial x} (-2xz) - \frac{\partial}{\partial y} (x^2y) \right) \\ &= (2z + 2x)\mathbf{i} - (0 - 0)\mathbf{j} + (-2z - x^2)\mathbf{k} = \langle 2z + 2x, 0, -2z - x^2 \rangle \end{aligned}$$

$$\begin{aligned}
\mathbf{curl\ curl\ } A &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2z + 2x & 0 & -2z - x^2 \end{vmatrix} \\
&= \mathbf{i} \left(\frac{\partial}{\partial y} (-2z - x^2) - \frac{\partial}{\partial z} (0) \right) \\
&\quad - \mathbf{j} \left(\frac{\partial}{\partial x} (-2z - x^2) - \frac{\partial}{\partial z} (2z + 2x) \right) \\
&\quad + \mathbf{k} \left(\frac{\partial}{\partial x} (0) - \frac{\partial}{\partial y} (2z + 2x) \right) = \mathbf{0} \cdot \mathbf{i} - (-2x - 2)\mathbf{j} + \mathbf{0} \cdot \mathbf{k} = \\
&= (2x + 2)\mathbf{j} = \langle \mathbf{0}, 2x + 2, \mathbf{0} \rangle
\end{aligned}$$