Answer on Question #56122 – Math – Calculus

Let φ(x,y,z)=xy2z and A=xzi−xy2j+yz2k,find

∂^3/∂x^2∂z(φA)

2i+2j−5k

5i−k

4i−2j

i+j

Solution

$$(\phi\mathrm{A}) = \mathrm{xy^2z} \cdot (\mathrm{xz}; \mathrm{xy^2}; \mathrm{yz^2}) = (\mathrm{x^2y^2z^2}; \mathrm{x^2y^4}z; \mathrm{xy^3z^3})$$

$$\frac{\partial^3}{\partial x^2 \partial z}(\mathrm{x^2y^2z^2}) = \frac{\partial^2}{\partial x^2}(2\mathrm{x^2y^2z}) = 4\mathrm{y^2z}$$

$$\frac{\partial^3}{\partial x^2 \partial z}(\mathrm{x^2y^4}z) = \frac{\partial^2}{\partial x^2}(\mathrm{x^2y^4}) = 2y^4$$

$$\frac{\partial^3}{\partial x^2 \partial z}(\mathrm{xy^3z^3}) = \frac{\partial^2}{\partial x^2}(3\mathrm{xy^3z^2}) = 0.$$

Thus,

$$\frac{\partial^3}{\partial x^2 \partial z}\left(\phi\vec{\mathrm{A}}\right) = 4\mathrm{y^2z}\,\vec{\mathrm{i}} + 2y^4\vec{j}.$$

If we need the answer only with numbers we need to choose the point where we calculate $\frac{\partial^3}{\partial x^2 \partial z}\left(\phi\vec{\mathrm{A}}\right)$.

Also we can see that answers **2i+2j−5k** and **5i−k** are false, because k-th component of the vector $\frac{\partial^3}{\partial x^2 \partial z}\left(\phi\vec{\mathrm{A}}\right)$ is zero. And the answer **4i−2j** is false because j-th component of the vector $\frac{\partial^3}{\partial x^2 \partial z}\left(\phi\vec{\mathrm{A}}\right)$ is always greater or equal than zero:

$$2y^4 \geq 0.$$

So, only **i+j** can be answer to the problem.

Answer: i+j.