

Answer on Question #56115 – Math - Vector Calculus

Question

Simplify

$$(A+B) \cdot (B+C) \times (C+A)$$

- a). $B \times C$
- b). $2 \cdot A \times B$
- c). $2 \cdot A \times C$
- d). $A \times B \times C$

Solution

By the properties of cross product, calculate

$$\begin{aligned}(\vec{A} + \vec{B}) \cdot ((\vec{B} + \vec{C}) \times (\vec{C} + \vec{A})) &= (\vec{A} + \vec{B}) \cdot (\vec{B} \times \vec{C} + \vec{B} \times \vec{A} + \vec{C} \times \vec{C} + \vec{C} \times \vec{A}) \\ &= (\vec{A} + \vec{B}) \cdot (\vec{B} \times \vec{C} + \vec{B} \times \vec{A} + 0 + \vec{C} \times \vec{A}) \\ &= \vec{A} \cdot \vec{B} \times \vec{C} + \vec{A} \cdot \vec{B} \times \vec{A} + \vec{A} \cdot \vec{C} \times \vec{A} + \vec{B} \cdot \vec{B} \times \vec{C} + \vec{B} \cdot \vec{B} \times \vec{A} + \vec{B} \cdot \vec{C} \times \vec{A} \\ &= \vec{A} \cdot \vec{B} \times \vec{C} + 0 + 0 + 0 + 0 + \vec{B} \cdot \vec{C} \times \vec{A} = 2 \cdot \vec{A} \cdot \vec{B} \times \vec{C}.\end{aligned}$$

Answer. $2 \cdot \vec{A} \cdot \vec{B} \times \vec{C}.$