Question 5606

The rate of change of the function f(x)=secx+cosx is given by the expression secxtanx-sinx. Show that this expression can also be written as sinxtan(x)^2

First, lets find derivative of $f(x)$ and then show easy operations to solve the given task.
Remember, that

$$sec(x)=\frac{1}{\cos(x)};\ \tan(x)=\frac{\sin(x)}{\cos(x)} \quad (1)$$

The derivative

$$f'(x)=-\sin(x)+\frac{1}{\cos^2(x)}\cdot\sin(x) \quad (2)$$

According to (1)

$$sec(x)\cdot\tan(x)=\frac{\sin(x)}{\cos^2(x)} \quad (3)$$

Then, combining (2) and (3), we obtain: $f'(x)=sec(x)\tan(x)-\sin(x)$ (4)
Also, we can convert (2), using $\sin^2(x)+\cos^2(x)=1$:

$$f'(x)=\frac{-\sin(x)\cos^2(x)+\sin(x)}{\cos^2(x)}=\frac{\sin(x)[1-\cos^2(x)]}{\cos^2(x)}=\frac{\sin^3(x)}{\cos^2(x)}=\sin(x)\cdot\tan^2(x) \quad (5)$$

We have proved both equalities.