

Answer on Question #56028- Math - Calculus

The number of locusts (l) t days after an infestation is given by the equation

$$l = 5t^2 + 10t + 100$$

The area of grass left (a) in m^3 is given by the equation $a = 500/l$

Use composite functions to determine the rate of change of the area of grass on the sixth day.

Explain why there is a limit to the time over which these equations would be realistic.

Solution

Using the rule for differentiation of composite functions,

the function to determine the rate of change of the area of grass is given by

$$v(t) = a'(t) = \left(\frac{500}{5t^2 + 10t + 100} \right)' = 500 \cdot \left(-\frac{1}{(5t^2 + 10t + 100)^2} \right) \cdot (5t^2 + 10t + 100)' = \frac{-500(10t + 10)}{(5t^2 + 10t + 100)^2}.$$

The rate of change of the area of grass on the sixth day is given by

$$v(6) = \frac{-500(10 \cdot 6 + 10)}{(5 \cdot 36 + 10 \cdot 6 + 100)^2} = \frac{-500 \cdot 70}{(340)^2} = -\frac{700}{2312} = -\frac{175}{578} \approx -0.303$$

There is a limit to the time over which these equations would be realistic because $t > 0$ and

$$5t^2 + 10t + 100 \neq 0.$$