

Answer on Question #55955 – Math - Calculus

Question

Prove that $\int_0^3 \int_1^{\sqrt{4-y}} (x+y) dx dy = 241/60$

Solution

$$\int_0^3 \int_1^{\sqrt{4-y}} (x+y) dx dy = \int_0^3 \left(\frac{1}{2} x^2 + yx \right) \Big|_1^{\sqrt{4-y}} dy = \int_0^3 \left(\frac{4-y}{2} + y\sqrt{4-y} - \frac{1}{2} - y \right) dy = \frac{3}{2} \int_0^3 dy - \frac{3}{2} \int_0^3 y dy + \int_0^3 y\sqrt{4-y} dy$$

$$\frac{3}{2} \int_0^3 dy = \frac{3}{2} y \Big|_0^3 = \frac{9}{2}$$

$$-\frac{3}{2} \int_0^3 y dy = -\frac{3}{4} y^2 \Big|_0^3 = -\frac{27}{4}$$

$$\int_0^3 y\sqrt{4-y} dy = \int_{z=4-y}^1 (4-z)\sqrt{z} dz = 4 \int_1^4 z^{3/2} dz - \int_1^4 z^{5/2} dz = 4 \left(\frac{2}{3} z^{3/2} \Big|_1^4 \right) - \left(\frac{2}{5} z^{5/2} \Big|_1^4 \right) = \frac{56}{3} - \frac{62}{5}$$

Thus,

$$\int_0^3 \int_1^{\sqrt{4-y}} (x+y) dx dy = \frac{9}{2} - \frac{27}{4} + \frac{56}{3} - \frac{62}{5} = \frac{241}{60}$$