Answer on Question #55953 – Math – Calculus

Let $f(z) = \frac{2}{z^2-4z+3}$. Find the Laurent series expansion of f about $z = 0$ valid in the region $|z| < 1$.

Solution

$$f(z) = \frac{2}{z^2 - 4z + 3} = \frac{2}{(z-1)(z-3)} = \frac{1}{z-3} - \frac{1}{z-1} = \frac{1}{1-z} - \frac{1}{3}\frac{1}{1-\frac{z}{3}}$$

The fractions are sums of geometric progressions:

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$$

$$\frac{1}{1-\frac{z}{3}} = \sum_{n=0}^{\infty} \left(\frac{z}{3}\right)^n$$

$$f(z) = \frac{1}{1-z} - \frac{1}{3}\frac{1}{1-\frac{z}{3}} = \sum_{n=0}^{\infty} z^n - \frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{z}{3}\right)^n = \sum_{n=0}^{\infty} z^n - \sum_{n=0}^{\infty}\frac{z^n}{3^{n+1}} =$$

$$= \sum_{n=0}^{\infty}\left(1 - \frac{1}{3^{n+1}}\right)z^n = \sum_{n=0}^{\infty}(1 - 3^{-1-n})z^n$$