

Answer on Question #55779 – Math – Algebra

You have a coupon for your favorite clothing store for \$ 25 off any purchase of more than \$50. The store is also running a 20%- off sale on its entire inventory. Let x be the original price, $f(x)$ be the price with the \$ 25 coupon applied, and $g(x)$ be the price with the 20% discount applied.

- Write an expression for $f(x)$
- Write an expression for $g(x)$
- What would the expression $(f \circ g)(x)$ represent?
- What would the expression $(g \circ f)(x)$ represent?
- If the store allows you to apply both the 20% discount and the \$25- off coupon, does it matter which you apply first? How do you know ?

Solution

a. $f(x) = \begin{cases} x - 25, & \text{if } x > 50 \\ x, & \text{if } x \leq 50. \end{cases}$ is the price with the \$ 25 coupon applied.

b. $g(x) = x - 0.2 * x = 0.8x$ is the price with the 20% discount applied.

c. $(f \circ g)(x) = f(g(x)) = \begin{cases} 0.8x - 25, & \text{if } 0.8x \geq 50 \\ 0.8x, & \text{if } 0.8x < 50 \end{cases} =$
 $= \begin{cases} 0.8x - 25, & \text{if } x \geq 62.5 \\ 0.8x, & \text{if } x < 62.5. \end{cases}$

d. $(g \circ f)(x) = g(f(x)) =$

$$= 0.8 * \begin{cases} x - 25, & \text{if } x \geq 50 \\ x, & \text{if } x < 50 \end{cases} = \begin{cases} 0.8(x - 25), & \text{if } x \geq 50 \\ 0.8x, & \text{if } x < 50 \end{cases} =$$
$$= \begin{cases} 0.8x - 20, & \text{if } x \geq 50 \\ 0.8x, & \text{if } x < 50. \end{cases}$$

- e. If $x \leq 50$ then $(f \circ g)(x) = 0.8x$ and $(g \circ f)(x) = 0.8x$.
Thus, $(f \circ g)(x) = (g \circ f)(x)$.
If $50 < x < 62.5$ then $(f \circ g)(x) = 0.8x$ and
 $(g \circ f)(x) = 0.8x - 20$.

Thus, $(g \circ f)(x) < (f \circ g)(x)$.

If $x \geq 62.5$ then $(f \circ g)(x) = 0.8x - 25$ and $(g \circ f)(x) = 0.8x - 20$.

Thus, $(f \circ g)(x) < (g \circ f)(x)$.

So if you will buy for more than \$ 62.5, it will be cheaper for you to apply \$25- off coupon after 20%- off sale.

If you will buy for less than \$ 62.5 and more than \$ 50, it is better for you to apply 20%- off sale after \$25- off coupon.

In the case when you will buy for less than \$ 50 your \$25- off coupon will not change the price, so it does not matter which you apply first: the 20% discount or the \$25- off coupon.