Answer on Question #55624 – Math– Algorithms | Quantitative Methods

Question

In recording measured values, a digit is significant if and only if it affects the -------------- of the measurement.

A. absolute error

B. relative error

C. inherent error

D. truncation error

Solution

By the *error* we mean the difference between the true value and the approximate value.

Absolute error is the amount of physical error in a measurement, period.

Relative error shows how good a measurement is relative to the size of the thing being measured.

Inherent error is involved in the statement of a problem before its solution. The inherent error arises either due to the simplified assumptions in the mathematical formulation of the problem or due to the errors in the physical measurements of the parameters of the problem.

Truncation error is caused by using approximate formulae in computation or on replace an infinite process by a finite one that is when a function f(x) is evaluated from an infinite series for x after 'truncating' it at a certain stage.

The significance figures of a number are digits that carry meaning contributing to its measurement resolution.

Rules for determining *significant digits*:

1. All non-zero digits are significant.
2. Zeroes which occur between two nonzero digits are also significant.
3. Zeroes to the left of the left most nonzero digit are never significant.
4. Zeroes to the right of the right most nonzero digit are significant only if they come from the measurement.

For example, the numbers 124, 1.24, 0.124 and 0.0124, all have three significant digits.

If measurement is 0.5 m, then its unit of measurement is 0.1 m, possible error is 0.05 m and relative error is 0.1.

If measurement is 0.005 m, then its unit of measurement is 0.001 m, possible error is 0.0005 m and relative error is 0.1.

These two measurements show that the unit of measure and the possible error in all the cases are different. Nevertheless, the relative error is the same. These zeroes are not significant and they do not affect relative error. Hence one can conclude that a digit is significant if and only if it affects the relative error.

Lemma 1. If x is the approximate value of X correct to m significant digits, then

$$\left|\frac{X-x}{X}\right| < 10^{-m}$$

Lemma 2. If a number is correct to n significant figures, and the first significant digit of the number is α_m, then the relative error is $E_R < \frac{1}{\alpha_m 10^{n-1}}$

Answer: B. relative error.