Answer on Question #55518 – Math – Algorithms | Quantitative Methods

Question 1

If f(1) = 1, f(3) = 19, f(4) = 49 and f(5) = 101, find the Lagrange's interpolation polynomial of f(x).

a. P(x)=x 3 −x 2 +1

b. P(x)=x 3 –3x 2 −5x–4

c. P(x)=x 3 –3x 2 +5x–6

d. P(x)=2x 2 –3x+5x–6

Solution

The Lagrange's interpolation polynomial has the form

$$L(x)=\sum_{i=0}^{n} y_i l_i(x),$$

where

$$l_i(x)=\prod_{j=0,\, j\neq i}^{n} \frac{x-x_j}{x_i-x_j}=\frac{x-x_0}{x_i-x_0}\cdots\frac{x-x_{i-1}}{x_i-x_{i-1}}\cdot\frac{x-x_{i+1}}{x_i-x_{i+1}}\cdots\frac{x-x_n}{x_i-x_n}.$$

Thus, we obtain

$$P(x)=\frac{(x-3)(x-4)(x-5)}{(1-3)(1-4)(1-5)}f(1)+\frac{(x-1)(x-4)(x-5)}{(3-1)(3-4)(3-5)}f(3)+\frac{(x-1)(x-3)(x-5)}{(4-1)(4-3)(4-5)}f(4)+\frac{(x-1)(x-3)(x-4)}{(5-1)(5-3)(5-4)}f(5)=$$

$$=\frac{-1}{24}(x-3)(x-4)(x-5)+\frac{19}{4}(x-1)(x-4)(x-5)+\frac{-49}{3}(x-1)(x-3)(x-5)+\frac{101}{8}(x-1)(x-3)(x-4)=$$

$$=x^3-x^2+1.$$

Answer: a. $P(x)=x^3-x^2+1$.

Question 2

The first divided difference of f with respect to x i and x i+1 denoted by f[x i ,x i+1] is defined as

a. f[x i ,x i+1]=f[x i+1]−f[x i]x i+1 −x i

b. f[x i ,x i+1]=x i+1 −x i f[x i+1]−f[x i]

c. f[x i ,x i+1]=f[x i]−f[x i+1]x i+1 −x i

d. f[x i ,x i+1]=f[x i]−f[x i+1]f[x i+1]−x i

Solution

The first divided difference of f with respect to x_i and x_{i+1} denoted by f[x_i ,x_{i+1}] is defined as

$$f[x_i,x_{i+1}]=\frac{f(x_{i+1})-f(x_i)}{x_{i+1}-x_i}.$$

Answer: a. $f[x_i,x_{i+1}]=\dfrac{f(x_{i+1})-f(x_i)}{x_{i+1}-x_i}$.