Answer on Question #55333 – Math – Statistics and Probability

At the town fair, you can pay \$5 to toss a ring at a set of bottles. If you get a "ringer" on the small mouth bottle, you win \$35. If you get a "ringer" on the medium bottle, you win \$10. If you get a "ringer" on the large bottle, you get your \$5 fee back (that is, you break even). If you miss, you are out the \$5 you paid to play. Ryan is a good shot and his probability of getting a ringer on the small, medium, and large bottles is 10%, 10%, and 5%, respectively.

X -\$5 \$0 \$10 \$35

P 0.75 0.10 0.10 0.05

a. Find the math expectation of Ryan's winnings for a single game.

b. Find the math expectation of Ryan's winnings after 5 games.

c. Find the variance of Ryan's winnings for a single game.

d. Find the standard deviation of Ryan's winnings for a single game.

e. Does it pay for Ryan to play this game at the fair?

f. Find the cumulative distribution function of Ryan's winnings for a single game and draw its graph.

Solution

a. $E[X] = (0.75)(-\$5) + (0.05)(\$0) + (0.10)(\$5) + (0.10)(\$30) = -\$0.25.$

b. Playing 5 games, he can expect to lose a quarter per game, which would be a loss of \$1.25:

$$E[5X] = 5E[X] = -\$0.25 \cdot 5 = -\$1.25,$$

c. The variance of Ryan's winnings for a single game is

$$Var = E[X^2] - (E[X])^2.$$

$E[X^2] = (0.75)(-5)^2 + (0.05)(0)^2 + (0.10)(5)^2 + (0.10)(30)^2 = 111.25.$

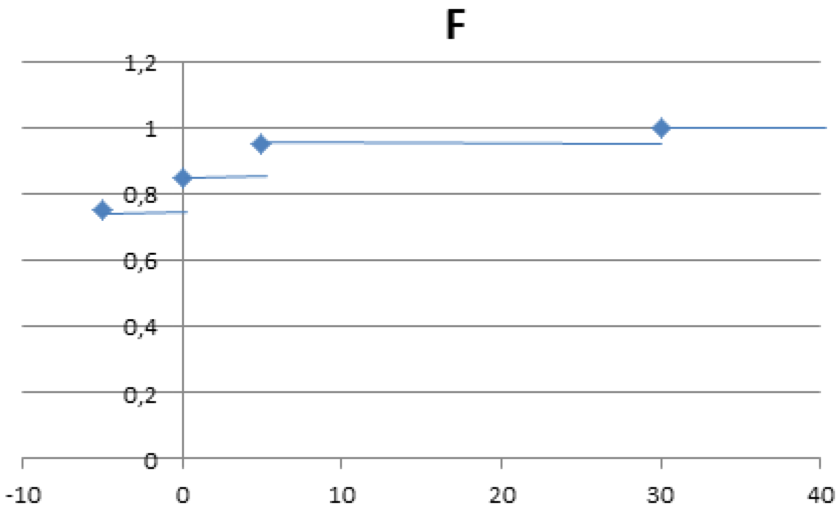
$$Var = 111.25 - (-0.25)^2 = 111.1875.$$

d. $SD = \sqrt{Var} = \sqrt{111.1875} = \$10.54.$

e. It's not, as Ryan stands to lose money overall.

f. The cumulative distribution function of Ryan's winnings for a single game is

X	-\$5	\$0	\$10	\$35
F(x)	0.75	0.75+0.10=0.85	0.85+0.10=0.95	0.95+0.05=1


F
1,2
1
0,8
0,6
0,4
0,2
0
-10
0
10
20
30
40