Answer on Question #55259 – Math – Statistics and Probability

According to The World Bank, only 9% of the population of Uganda had access to electricity as of 2009. Suppose we randomly sample 150 people in Uganda. Let X = the number of people who have access to electricity.

a. What is the probability distribution for X?

b. Using the formulas, calculate the mean and standard deviation of X.

c. Use your calculator to find the probability that 15 people in the sample have access to electricity.

d. Find the probability that at most ten people in the sample have access to electricity.

e. Find the probability that more than 25 people in the sample have access to electricity.

Solution

a. In this case we are given sample size n and probability p so this is a binomial distribution with $n = 150$ and $p = 0.09$.

b. The mean of X is

$$\mu = np = 0.09 \cdot 150 = 13.5.$$

A standard deviation is

$$\sigma = \sqrt{np(1-p)} = \sqrt{0.09 \cdot 150(1-0.09)} = 3.5.$$

c. The probability that 15 people in the sample have access to electricity

$$binomialpdf(150; 0.09; 15) = \frac{150!}{15!\,(150-15)!}0.09^{15}(1-0.09)^{50-15} = 0.0988.$$

In Excel 2007 and earlier it is calculated by

$$= BINOMDIST(15; 150; 0{,}09; FALSE)$$

In Excel 2010 and later it is calculated by

$$= BINOM.DIST(15; 150; 0{,}09; FALSE)$$

d. The probability that at most ten people in the sample have access to electricity is the cumulative binomial probability with n=150, p=.09, and x=10 is

$$binomialcdf(150; 0.09; 10) = \sum_{k=0}^{10} \frac{150!}{k!\,(150-k)!}0.09^{k}(1-0.09)^{50-k} = 0.1987.$$

In Excel 2007 and earlier it is calculated by

$$= BINOMDIST(10; 150; 0{,}09; TRUE)$$

In Excel 2010 and later it is calculated by

$$= BINOM.DIST(10; 150; 0{,}09; TRUE)$$

e. The cumulative binomial probability of having at most 25 people have electricity (n=150,p=.09, x=25) is

$$binomialcdf(150; 0.09; 25) = \sum_{k=0}^{25} \frac{150!}{k!\,(150-k)!} 0.09^k (1-0.09)^{50-k} = 0.9991.$$

In Excel 2007 and earlier it is calculated by

$$= BINOMDIST(25; 150; 0{,}09;\ TRUE)$$

In Excel 2010 and later it is calculated by

$$= BINOM.DIST(25; 150; 0{,}09; TRUE)$$

So the probability of having more than 25 people is

$$1 - 0.9991 = 0.0009.$$