Answer on Question #55138 – Math – Statistics and Probability

The following data were obtained from two random samples. Test whether the samples come from the same normal population at 5% level of significance.

No. Size Mean Sum of squares of deviation from mean

1 10 15 90

2 12 14 108

Solution

$$s_1 = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n_1 - 1}} = \sqrt{\frac{90}{10 - 1}} = \sqrt{10} \approx 3.16.$$

$$s_2 = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n_2 - 1}} = \sqrt{\frac{108}{12 - 1}} = 3.13.$$

1. Test whether means are different:

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$

$$T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{15 - 14}{\sqrt{\frac{3.16^2}{10} + \frac{3.13^2}{12}}} = 0.742.$$

Critical value for Student's T-distribution with $10 + 12 - 2 = 20$ degrees of freedom and $\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$ is

$$t^* = 2.086.$$

$T < t^*$, so we can conclude that means are the same.

2. Test whether variances are different:

$H_0: \sigma_1^2 = \sigma_2^2$

$H_a: \sigma_1^2 \neq \sigma_2^2$

$$F = \frac{s_1^2}{s_2^2} = \frac{3.16^2}{3.13^2} = 1.019.$$

Critical value for F-distribution with $10 - 1 = 9$ and $12 - 1 = 11$ degrees of freedom, $\alpha = 0.05$, is

$$f^* = 3.59.$$

$F < f^*$, so we can conclude that variances are the same.

Answer: the samples come from the same normal population.